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THE BEHAVIOR COMPARISON BETWEEN MEAN REVERSION AND JUMP DIFFUSION OF CDS SPREAD[†]

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Abstract

This paper empirically investigated the behavior of Korean CDS spread which captures the excess kurtosis and heavier tails (i.e. leptokurtic behavior). In capturing the dynamics of the Korean CDS spread, this study notably focuses on the comparison of mean reverting drifts and jump part of the continuous-time models of CDS spread. The results are as follows. First, Empirical findings indicate that the addition of jumps leads to a lower expected return and volatility. This result implies that jumps account for a substantial portion of the overall volatility of the return data. Second, During Pre and Post Crisis period the GBM is better than competing models in terms of parameter significance, log-likelihood and the BIC. Third, the addition of jumps improves performance significantly since all jump diffusion processes outperform their diffusion counterparts especially during the crisis period. Finally, the addition of mean-reversion appears to increase the goodness-of-fit, especially in the case of the jump-diffusion models during the crisis period.

Keywords: Excess Kurtosis, CDS, GBM, Mean Reverting, Jump Diffusion

1. Introduction

The finance literature has considered different models for asset-price dynamics. Log-returns are normally distributed with constant volatility. However, this does not capture the excess kurtosis of log-returns. It is well known that asset return distributions are heavy tailed and skewed, which are at odds with the classical geometric Brownian motion models. Many studies have been undergone to propose modifications to explain these phenomena such as a leptokurtic feature, a volatility smile, and a volatility clustering effect, i.e. (1) skewed distribution with higher peak and heavier tails (i.e. leptokurtic behavior) of the return distribution and (2) the volatility smile. The frontier works can be divided into continuous-time models and discrete-time models. Examples of the former include Merton (1976), Hull and White (1987) and Bates (1996a, 1996b), and of the latter the ARCH models of Engle (1982) and Bollerslev (1986). The descriptive statistics sometimes shed doubts on the validity of the standard Brownian motion process of continuous-time models. A number of studies connect the kurtosis with jump risk (Drost *et al.*

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1998; Bates, 1996a, 1996b). This excess kurtosis is accounted for by a jump-diffusion model like Merton's and others. Motivated by these finding this study examined the ability of mean reversion, various popular diffusion and jump diffusion continuous-time processes in capturing the dynamics of the Korean CDS spread. This study notably focuses on the comparison of mean reverting drifts and jump part of the continuous-time models of CDS spread.

The remainder of the paper is organized as follows: Section 2 describes data and the descriptive statistics. Section 3 presents a model of general diffusion model like GBM, mean reversion Vasicek and CIR model, diffusion model with jump. Section 4 illustrates methodology and provides the results of the empirical analysis. Section 5 provides some concluding remarks.

2. Data and Descriptive Statistics

A credit default swap is an agreement between two parties to exchange the credit risk of a reference entity. The buyer of the CDS is said to buy protection, has a similar credit risk position to selling a bond short and investing the proceeds in a risk-free asset. Conversely, the seller of the CDS is said to sell protection and selling protection has a similar credit risk profile to maintaining a long position in a bond or a loan. If a credit event occurs, the compensation is to be paid by the protection seller to the buyer via either physical settlement (i.e., receiving the defaulted bond against payment of par) or cash settlement (i.e., paying the difference between par and the bond's recovery value), as specified in the contract. The premium paid by the protection buyer to the seller, often called "CDS spread," is quoted in basis points per annum of the contract's notional value, is usually paid quarterly, and is not based on any specific risk-free bond or benchmark interest rate.

We collect data on the quotes for sovereign CDS spreads with a 5-year tenure from a comprehensive database compiled by Bloomberg and the Markit Group. We used level series and return series in daily quotes of Korean sovereign CDS indices with a maturity of five years from January 3, 2006 to June 5, 2014 (Figure 1). On August 9, 2007, BNP Paribas announced the freeze of redemptions for three of its investment funds, citing an inability to value them. The dollar shortage of European financial institutions led to a global dollar shortage, as Lehman Brothers failed on September 15, 2008, and the Federal Reserve Board (FRB) announced a bailout package for American Insurance Group (AIG). The Korean FX markets were also not immune to the crisis. Therefore, three periods were divided under this study, i.e. Pre-crisis period (January, 2006 to August 2007), crisis period (August, 2007 to December, 2009), and post-crisis period (January, 2010 to May, 2014). Specifically, we employ a continuous-time model to estimate the featured behavior of Korean Sovereign using daily 5-year CDS indexes.

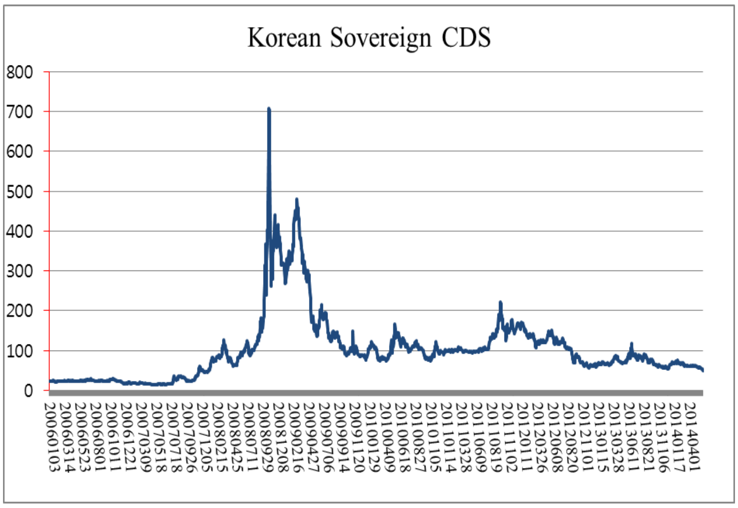

Figure 1. The Behavior of Korean Sovereign CDS Spread

Notes: The datasets used this paper consist of daily closing prices for the period 3/1/2006–5/6/2014.

Table 1 gives descriptive statistics for the daily quotes of CDS indices. The CDS level series show a positive mean and skewness, also the excess kurtosis shows that all series are leptokurtic (i.e. fat tails). The CDS return series also show the excess kurtosis of log-returns. Asset return distributions are heavy tailed and skewed, which are at odds with the classical geometric Brownian motion models. This excess kurtosis is accounted for by a jump-diffusion model. The skewness and kurtosis coefficients suggest a leptokurtic distribution with positively skewed returns. This is also confirmed by the Jarque-Bera test results which clearly reject the null hypothesis of a normal distribution for both levels and returns. The stationarity properties of the data were examined through three unit root tests table.

Table 1. Descriptive Statistics of CDS Prices and Logarithmic Returns

	CDS Prices	**CDS Returns**
Mean	101.0678	1.0001
Median	85.9700	0.0000
Maximum	708.6005	111.404
Minimum	14.0000	0.9169
Std. Dev.	85.2455	0.0105
Skewness	2.3810	0.5836
Kurtosis	10.5732	12.2199
Jque-Bera	7146.193*	7712.172*
Q(20)	37,100.00 (0.00)	56.931 (0.00)
ADF	-2.5267	-44.6896*
PP	-2.6045	-44.7829*

Notes*:* The statistics are based on daily sampled data from January 3, 2006, to June 5, 2014 (2,143 observations for each series). Q(20) is the Ljung-Box statistic for serial correlation. ADF refers to the Augmented Dickey–Fuller test (Dickey and Fuller, 1979), PP to the Philips-Peron test (Phillips and Perron, 1988). The lag structure in the ADF test is selected automatically on the basis of the Bayesian Information Criterion (BIC). The values in parentheses are p-values. * indicates the rejection at the 1% level.

The results presented in Table 1 suggest that at conventional significance levels, CDS indexes are non-stationary, and this result somewhat contradicts the common finding of mean reverting behavior observed in commodities and energy (Schwartz, 1997), and investment asset, such as equity and bond. Thus, Korean CDS spread are considered to be considered to be commodities for consumption like EUAs (European Union Allowance).

3. Continuous-Time Dynamics of Korean CDS Prices

3.1. Geometric Brownian Motion (GBM) and Mean Reversion

$$ds_t = \mu s_t \ + \ \delta s_t \mathrm{dw_t} \quad (1)$$

Here w is a standard Brownian motion, a special diffusion process that is characterized by independent identically distributed (iid) increments that are normally (or Gaussian) distributed with zero mean and standard deviation equal to the square root of the time step. The GBM process matches the first two unconditional moments, but cannot match the higher moments of the empirical distribution of the asset price. GBM have been very popular for modeling the evolution of stocks, (e.g. Merton, 1976; Osborne, 1959). This model further requires an assumption of perfectly divisible assets and a frictionless market. However, it has been discovered that the stock price does not fully fulfill the assumption of classical Brownian motion, since negative evidence is found such as non-Markovian/non-semi martingale features and characterized by irregular long cycles and long-term memory. To cope these problems the fractional brown motion model is introduced and applied in stock (Rostek and Schobel, 2013;

Potgieter, 2009), weather (Benth, 2003), foreign exchange (Sun *et al.* 2013; Meng and Wang, 2010; Xiao *et al.* 2010), energy and electric markets (Cheung and Lai, 1993; Diebold *et al.* 1991).

Mean reversion is a tendency for a stochastic process to remain near, or tend to return over time to a long-run average value. For example, interest rates exhibit mean reversion. The mean reversion of the interest rate and credit spread indices is supported by the intuition that they are linked to the economic cycle and arbitrage opportunity. In theory, CDS spreads should be closely related to bond yield spreads. Define y as the yield on an n-year par yield bond issued by a reference entity, r as the yield on an n-year par yield riskless bond (interest swap), and s as the n-year CDS spread. The relationship:

$$\mathrm{s} = \mathrm{y} - \mathrm{r}$$

should therefore hold approximately. If s is greater than y – r, an arbitrageur will find it profitable to buy a riskless bond, short a corporate bond and sell the credit default swap. If s is less than y-r, the arbitrageur will find it profitable to buy a corporate bond, buy the credit default swap and short a riskless bond. Therefore CDS spread sometimes equals spread over a benchmark Treasury bond, or spread over swap rate (Asset swap spread). If the difference between the CDS premium and the asset swap spread were to diverge from zero that would constitute a theoretical arbitrage opportunity. Previous studies (Blanco *et al.* 2005; Chan-Lau and Kim, 2004; Norden and Weber, 2009; Zhu, 2005) have found that this long-term theoretical equilibrium relationship broadly holds, though they have also shown that short-term deviations can be considerable due to cheapest to deliver (CTD) option, repo cost of bond, and unequal maturity between two financial products. Therefore previous studies find a zero long-run basis anda stationary basis between two spread for entities. If the tests find AR features, then the process is considered as mean reverting.[1]

The following mean reversion model has dynamics as parameters in the model are time-dependent. The most commonly accepted hierarchy one-factor model in each case has the Vasicek model (1977, a short-rate model; θ and α constant), Cox *et al.* (1985, CIR square-root process), the Hull-White model (1990, θ has time t dependence), and the extended Vasicek model (θ and α also time dependent).[2] Credit spread and Credit Default Swap (CDS) spread has appeared to be mean reverting. This assumption also has some support from the econometric analysis.

$$\mathrm{d}r_t = (\theta_t - \alpha_t r_t)dt + \delta d\mathrm{w}_\mathrm{t}$$

The instantaneous spot rate (or short rate) follows an Ornstein-Uhlenbeck process with constant coefficients under the statistical or objective measure used for historical estimation. This model assumes a mean-reverting stochastic behavior of interest rates.[3] Vasicek model is

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[1] If fat tails are not found and the AR test failed, this means a process lacking mean reversion but with Gaussian shocks, it could be modeled as an arithmetic Brownian motion. In a way, there could be some sort of mean reversion even under non-Gaussian shocks. It could be modeled as jump-extended HW (Vasicek) or exponential HW (Vasicek) models with mean reversion mixed with fat tails. They are used with fatter tails of the Gaussian distribution and possibly nonlinear behavior.

[2] The main drawback of one-factor models is the lack of empirical realism as they do not fit accurately the current rate and consequently, do not price correctly fixed income assets. In order to cope with this problem, two-factor models such as, for instance, Longstaff and Schwartz (1992).

[3] The Vasicek model has a major shortcoming that is the non null probability of negative rates. This is an unrealistic property for the modeling of positive entities like interest rates or credit spreads. One should point out that if this model is assumed for the short rate or stochastic default intensity, there is no closed form bond pricing or survival probability formula and all curve construction and valuation according to this model has to be done numerically through finite difference schemes, trinomial trees or Monte Carlo simulation. The historical calibration and simulation of the Exponential Vasicek model can be done by calibrating and simulating its log-process.

given by the following SDE:

$$\mathrm{dr_t} = \lambda(\theta - r_t)dt + \delta d\mathrm{w_t} \quad (2)$$

The parameter λ is the mean reversion speed from θ of the CDS price process r. The volatility parameter δ is positive. This model can be analyzed by considering Ornstein-Uhlenbeck process. Brigo and Mercurio (2006) apply Ito's formula to exponential function. Mean reverting square-root process (CIR) model is given by the following SDE:

$$\mathrm{dr_t} = \lambda(\theta - r_t)dt + \delta\sqrt{r_t}d\mathrm{w_t} \quad (3)$$

Models (2) and (3) also have been widely used in the equity mutual fund (Kellerhals and Schobel, 2002), commodity (Schwartz, 1997; Schwartz and Smith, 2000; Manoliu and Tompaidis, 2002), interest rate and volatility modeling (Windcliff *et al.* 2006; Detemple and Osakwe, 2000; Pan, 2002), and energy price modeling (Hahn *et al.* 2014; Kobari *et al.* 2014; Meade, 2010; Pindyck, 2001; 1999) literature. However, it has been discovered that the interest rate and does not fully fulfill the assumption of classical mean reverting Brownian motion, since non-Markovian and long memory features be characterized. The use of fractional brown mean reverting model is introduced in interest rate (Laurini and Hotta, 2013; Meade and Maier, 2013; Biagini *et al.* 2013), equity warrant (Xiao, 2014), and energy (Sun *et al.* 2013).

3.2. Models with Jump

The descriptive statistics discussed in Table 1 shed doubts on the validity of the standard Brownian motion process assumptions. We examined the ability of various popular diffusion and jump diffusion continuous-time processes in capturing the dynamics of the CDS prices (Chan *et al.* 1992; Dotsis *et al.* 2007) such as in the context of interest rates and implied volatility, respectively. Uncovering the continuous-time dynamics of CDS prices is generally a necessary step for choosing the appropriate CDS price behavior. As you see the Figure 1, Korea Sovereign CDS spread had jump behavior during the Global financial crisis of 2009. Zhang *et al.* (2005, 2009) assert that the relationship between CDS and jump risk is nonlinear. Chen *et al.* (2008) regard kurtosis as a proxy of jump risk by taking nonlinear relationships into account, they employ a copula technique to measure the dependence structure between the CDS return and the corresponding tendency of jumps.[4]

A jump diffusion process is leptokurtic and can be skewed. Merton (1976, 1990) added Poisson jumps to a standard GBM process to approximate the movement of asset prices subject to occasional discontinuous breaks. Compared with the normal distribution of the returns in the GBM model, the log-returns of a GBM model with jumps are often leptokurtotic. In a jump-diffusion model with compound Poisson jump. This model can be subject to idiosyncratic shocks modeled through jumps.

$$ds_t = \mu s_t + \delta s_t \mathrm{dw_t}$$

Suppose that a jump occurs in [t; t + dt] with probability λdt along with the usual GBM, the asset jumps, i.e. $S \rightarrow yS$, where y is a jump size. We will restrict y to be non-negative. If we have a combination of GBM and a rare jump event, then under the real probability measure P,

[4] This result is economically intuitive and consistent with Abid and Naifar (2005), Zhang *et al.* (2005), and Colin-Dufresne *et al.* (2001). The dependence structure is positive, indicating that protection sellers ask for higher CDS return to compensate for a higher tendency of jumps.

the candidate processes considered are nested in the following stochastic differential equation (SDE) in Merton (1976) is given by:

$$\mathrm{d}s_t = \mathrm{d}s_t\mathrm{dt} + \mathrm{d}s_t\mathrm{dw_t} + s_t(\mathrm{y}-1)\mathrm{d}n_t \tag{4}$$

where s_t is the CDS spread at time t, *w* is a standard Wiener process, μs_t is the drift,$\sigma s_t$is the diffusion coefficient, and$y$is the jump amplitude. The drift, the diffusion and the jump coefficients are assumed to be general functions of time and the CDS spread. The compound Poisson process has logarithmic normal distributed jumps. The process n_t is a standard Poisson process which models the number of jumps and has intensity λ > 0. *y* is a jump size distribution which is log-normal distribution. *n, y* and *w* are independent with the log-normal assumption for the jumps. The jump component is controlled by a Poisson process$\mathrm{q_t}$ is with constant arrival parameter λ, i.e. Pr{$\mathrm{dq_t}$= 1} = $\lambda\mathrm{dt}$ and Pr{ $\mathrm{dq_t}$=0} = 1−λdt. $\mathrm{dw_t}$, $\mathrm{dq_t}$and y are assumed to be mutually independent. Several different models can be obtained by combining various assumptions for the components of $\mu(\mathrm{r_t},\mathrm{t})$, $\sigma(\mathrm{r_t},\mathrm{t})$, and y($\mathrm{r_t}$,t). Average mean of jump (μ_J) determine skewness of distribution. Jump size volatility (σ_J) controls the peakedness of the density, it is peaked for small values, but flat for large values. Jump intensity λ leads the peakedness of distribution, it is peaked and more concentrated around the mean value for decreasing values.

$$\text{Mean}\, c_1 = \mathrm{E}[\mathrm{X(t)}] = \mathrm{t}(\mathrm{r}-\mathrm{d}+\lambda\mu_j)$$
$$\text{Variance}\;\; c_2 = \mathrm{V}[\mathrm{X(t)}] = \mathrm{t}\left(\sigma^2 + \lambda\sigma_J^2 + \lambda\mu_j^2\right)$$
$$\text{Skewness}\; c_3 = \mathrm{t}\lambda\left(3\sigma_J^2\mu_j + \mu_j^3\right)$$
$$\text{Kurtosis}\, c_4 = \mathrm{t}\lambda\left(3\sigma_J^3 + \sigma\mu_j^2\sigma_J^2 + \mu_J^4\right)$$

As mean of jump size (μ_J) and jump size volatility (σ_J) increase, average jump size and the uniformity of jump sizes increases. By adding jumps to the archetypal price process, two well-known finite jump models of Merton (1990), with lognormal jumps and Kou (2002), with double exponential distributed jumps are both able to partly explain the observed deviations from the benchmark model which are characterized by the leptokurtic feature and volatility smile. But Carr *et al.* (2002) propose the more general approach with possibly infinite jump activity. Jump diffusions are particularly tractable if the distribution of the jumps is a combination (or a mixture) of exponential distributions. Merton's jump-diffusion model with the log-normal distribution is better able to produce the volatility smile phenomenon. However, the use of the log-double-exponential distribution instead of the log-normal distribution allows the introduction of asymmetric leptokurtic features. Kou (2002) argues that this model better matches empirical data without adding much complexity to the model.[5]

To adequately model the extreme return outliers in equity markets, more recent theoretical and empirical research also argues in favor of jumps in the price process,[6] for example, currency options (Xiao *et al.* 2010), double-jump currency option model (Xu *et al.* 2013; Guo and Hung, 2007), commodity prices (Eydel and and Geman, 1998; Schwartz and Smith, 2000; Escribano *et al.* 2002), option pricing (Merton, 1976; Duffie *et al.* 2000), risk

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[5] The downside of moving to the log-double-exponential distribution is that it uses more parameters than the log-normal distribution. Moreover, many theoretical results are only valid for the log-normal distribution. A solution to a jump-diffusion model can be obtained by solving a partial integro-differential equation (PIDE) due to the integral term the discretization leads to a full matrix.

[6] Bates (1996b) proposes to combine both stochastic volatility and jumps only in the value of the underlying asset, and Duffie *et al.* (2000) augment this model by additional discontinuities in the stochastic variance process. More general jump-diffusion models with stochastic volatility are considered in Levy process with Phase-type jumps in both directions, a double exponential jump -diffusion model (Kou, 2002).

management (Duffie and Pan, 2001), and asset allocation (Jarrow and Rosenfeld, 1984; Liu *et al.* 2003).

GBM augmented by jumps (GBMJ):

$$ds_t = (\mu - \lambda\mu_j)s_t dt + \sigma s_t dw_t + (y-1)dq_t \quad (5)$$

MRS (CIR) augmented by jumps (MRSJ):

$$ds_t = \kappa(\theta - \lambda\mu_j)s_t dt + \sigma s_t dw_t + (y-1)dq_t \quad (6)$$

Processes (5) and (6) have been very popular for modeling the evolution of stocks, commodity prices and indices (e.g. Merton, 1976; Schwartz, 1997; Bierbrauer *et al.* 2007) and have proportional structure with μ being the expected return of the asset per unit of time and σ its volatility, while the other four (three) have mean reverting drifts. The model (6) has been employed, among others, by Bakshi *et al.* (2003) and Dotsis *et al.* (2007) for describing the dynamics of the instantaneous and implied volatility. In equations (2) – (4) and (6), K is the speed of mean reversion, θ is the unconditional long-run mean, and r the volatility of the asset price. In equation (6), λ is the average number of jumps per year, and *y* is the jump size, and μ_j, σ_j are the mean and the standard variation distributed normally according to the density function, respectively. Finally, in equation (6) the jump size is drawn from an asymmetric double exponential distribution: with *p, q* ≥ 0 and *p* + *q* = 1, representing the jump probabilities and $1/\boldsymbol{n_1}$, $1/\boldsymbol{n_2}$ the mean sizes of the upward and downward jumps respectively (see Kou, 2002; Dotsis *et al.* 2007 for more details).

4. Methodology and Empirical Findings

For the estimation of parameters, we used maximum likelihood estimation (usually abbreviated to MLE). MLE can be used for both continuous and discrete random variables. This is done by differentiating the Gaussian density function with respect to each parameter and setting the resulting derivative equal to zero. We refer to Dotsis *et al.* (2007) in calculating MLE. The conditional MLE to find the parameter estimates uses the conditional transition density function $\mathrm{f}[S(t+\tau)|S(t),\ \Theta](\tau > 0)$ of the process S_t and Θ which is the set of parameters to be estimated. The maximal log-likelihood value L is given by the following:

$$\mathrm{L} = \max \sum_{t=1}^{T-t} \log(\mathrm{f}\,[S(t+\tau)|S(t),\Theta])$$

We follow the approximation method which is described by Ait-Sahalia (2002). Ait-Sahalia generated accurate closed-form approximations to the transition functions of diffusion processes. For Geometric Brownian motion process with jump which the logarithm of the jump size of y is distributed normally with mean and variance γ and δ^2, Press (1967) suggested that the probability density function of log returns $x_t = \log\frac{S(t+\tau)}{S(t)}$ is described as a discrete Poisson mixture of j normal probability density functions where j tends to infinity, i. e.

$$\mathrm{f}[S(t+\tau)|S(t),\Theta] = \sum_{j=0}^{\infty} \frac{(\lambda\tau)^j e^{-j\tau}}{j!} \frac{1}{\sqrt{2\pi(\sigma^2\tau + j\delta^2)}} \exp\left(-\frac{(x_t - \alpha\tau - j\gamma)^2}{2(\sigma^2\tau + j\delta^2)}\right)$$

Where, j is the number of jumps and $\alpha = \mu - \frac{1}{2}\sigma^2$.

But, the conditional density of the mean-reverting square-root process with jump is derived from the characteristic functions $\mathrm{F}(S_t, \tau; u) = E(e^{iuS_{t+\tau}}|S_t;\, \Theta)$. Duffie *et al.* (1998) proved that for affine diffusion and jump diffusion processes, the characteristic function is:

$$\mathrm{F}(S_t, \tau; u, \Theta) = \exp(A(\tau; u) + B(\tau; u)S_t).$$

By the Fourier inversion of characteristic functions, the conditional density is:

$$\mathrm{f}[S(t+\tau)|S(t), \Theta] = \frac{1}{\pi}\int_0^{\infty} Re\left[e^{-iuS(t+\tau)}F(S(t), \tau; u, \Theta)\right]$$

In calculating numerical integration we used Gauss-Kronrodquadratures. In computing the conditional density of the mean-reverting square-root process with jump, Bakshi and Cao (2004) showed the formulas below:

$$\mathrm{A}(\tau; u) = \alpha(\tau, u) + z_1(\tau, u) + z_2(\tau, u)$$
$$\mathrm{B}(\tau; u) = \frac{kuie^{-k\tau}}{k - \frac{1}{2}i\sigma^2 u(1 - e^{-k\tau})}$$

Where:

$$\alpha(\tau; u) = -\frac{2k\theta}{\sigma^2} \times \log\left(\frac{k - \frac{1}{2}i\sigma^2 u(1 - e^{-k\tau})}{k}\right)$$
$$z_1(\tau; u) = -\frac{2\lambda p}{2k - \eta_1} \times \log\left(\frac{k - \frac{1}{2}i\sigma^2 u + iu\left(\frac{\sigma^2}{2} - \frac{k}{\eta_1}\right)e^{-k\tau}}{k - \frac{iuk}{\eta_1}}\right)$$
$$z_2(\tau; u) = -\frac{2\lambda(1-p)}{2k + \eta_2\sigma^2} \times \log\left(\frac{k - \frac{1}{2}i\sigma^2 u + iu\left(\frac{\sigma^2}{2} + \frac{k}{\eta_2}\right)e^{-k\tau}}{k + \frac{iuk}{\eta_2}}\right)$$

The parameters of the models under consideration were estimated in MATLAB using a Maximum Likelihood (ML) approach with CDS spread data from Markit. Performance was evaluated in terms of both statistical accuracy and model parsimony. Table 2, 3 and 4 present the estimated parameters and corresponding t-statistics (the latter in brackets), the log-likelihood values (3) and the Bayesian Information Criterion (BIC) for each one of the models under scrutiny for the whole sample under study. The estimation results for the pre-crisis, crisis, and post-crisis period subsample are reported in Tables 2, 3, and 4 respectively.

Table 2. Results of Continuous-Time Models – Pre-Crisis Period: 03/01/2006 to 31/08/2007

Parameter	GBM (1)	Vasicek (2)	CIR (3)	Merton Jump (4)	GBM +Jump (5)	CIR Jump (6)
μ	0.3577 (0.771)			0.3230 (0.7015)	0.3073 (0.7166)	
κ		1.1534 (0.2121)	0.5147 (10.9046)			0.0123 (2.1206)
θ		0.2719 (0.7388)	0.3202 (1.6897)			0.6103 (25.4219)
σ	0.5583 (7.049)	0.1458 (5.7628)	0.5595 (7.0288)	0.5580 (7.0543)	0.3453 (11.5872)	
λ				30.6019 (10.6452)	13.2914 (2.3042)	10.0094 (3.9274)
μ_j				0.0011 (11.000)	0.1024 (2.9257)	
σ_j				0.001 (10.000)	0.0369 (3.8842)	
ρ						0.6431 (8.2448)
$1/n_1$						0.0002 (20.000)
$1/n_2$						0.0012 (4.0000)
3	1435.12	1350.16	1435.44	795.05	907.94	639.02
BIC	-2865.01	-2692.47	-2863.03	-1579.02	-1802.79	-1259.72

Notes: Numbers in brackets are t-statistics. The log-likelihood value (I) and the Bayesian Information Criterion (BIC) are also reported.

Table 3. Results of Continuous-Time Models – Crisis Period: 01/09/2007 to 31/12/2009

Parameter	GBM (1)	Vasicek (2)	CIR (3)	Merton Jump (4)	GBM +Jump (5)	CIR Jump (6)
μ	0.828 (1.370)			0.8535 (1.2463)	0.0098 (0.00162)	
κ		2.2285 (0.6872)	0.6938 (1.2584)			0.0653 (1.3604)
θ		1.7413 (1.9602)	0.6881 (0.9806)			0.4978 (20.7416)
σ	0.9130 (13.4045)	2.5491 (7.8632)	0.9139 (17.3401)	0.9111 (17.3542)	0.9111 (17.3854)	
λ				34.642 (4.6632)	17.5402 (1.3323)	26.6912 (3.1236)
μ_j				-0.0008 (-0.1777)	0.0008 (0.8000)	
σ_j				0.0011 (11.000)	0.0012 (6.000)	
ρ						0.6076 (1.1123)
$1/n_1$						0.0012 (12.000)
$1/n_2$						0.4499 (1.4282)
3	708.21	717.63	707.63	830.80	829.88	973.55
BIC	-1410.89	-1426.91	-1406.97	-1647.79	-1645.94	-1927.76

Notes: Numbers in brackets are t-statistics. The log-likelihood value (I) and the Bayesian Information Criterion (BIC) are also reported.

Table 4. Results of Continuous-Time Models – Post Crisis: 1/1/2010 to 5/6/2014

Parameter	GBM (1)	Vasicek (2)	CIR (3)	Merton Jump (4)	GBM Jump (5)	CIR Jump (6)
μ	0.0662 (0.2219)			0.0600 (0.2020)	0.0085 (0.0298)	
κ		0.5883 (3.5269)	0.5450 (2.0540)			0.2566 (3.5393)
θ		0.3254 (0.6195)	0.3107 (1.9443)			0.1541 (0.3215)
σ	0.6131 (26.089)	0.7077 (18.7788)	0.6123 (26.1667)	0.6128 (26.0765)	0.6129 (26.0808)	0.6419 (22.8434)
λ				6.8763 (6.1079)	9.3806 (5.6037)	11.2282 (0.6720)
μ_j				0.0015 (5.000)	0.0011 5.5000	
σ_j				0.0002 (20.000)	0.0001 10.000	
ρ						0.8188 (7.3699)
$1/n_1$						0.0049 (1.9609)
$1/n_2$						0.0015 (0.3846)
3	3080.40	1852.53	2080.50	2007.35	2007.33	1992.24
BIC	-4154.72	-3695.94	-4151.88	-3990.50	-3999.46	-3963.20

Notes: Numbers in brackets are t-statistics. The log-likelihood value (l) and the Bayesian Information Criterion (BIC) are also reported.

A comparison between the performing diffusion, Mean reversion, and jump-diffusion process, respectively indicates that the addition of jumps leads to a lower expected return (μ) and volatility (σ). This result implies that jumps account for a substantial portion of the overall volatility of the return data. Second, A first observation is that the ranking of the processes during pre and post crisis period remains almost the same, with the GBM continuing to dominate the competing models. However, jumps included mean reversion (CIR) dominate the competing models in crisis period. Finally, it can be observed that the likelihood and BIC values demonstrate a larger variability for the models estimated over the pre and post crisis period when compared to that for the models estimated over the crisis period.

The results provide several interesting insights. First, during Pre and Post Crisis period in Korean CDS market we can observe that the GBM is better than competing models in terms of parameter significance, log-likelihood and the BIC. For the diffusion processes, GBM is slightly better than Vasicek and CIR model. CDS prices have a proportional structure that has no jumps which can be explained by standard diffusion processes during the stable period. Second, the addition of jumps improves performance significantly since all jump diffusion processes outperform their diffusion counterparts especially during the crisis period. The findings indicate that CDS prices have jumps, i.e., they are subject to large movements and shocks which could not be explained by standard diffusion processes during the crisis period. Third, the addition of mean-reversion appears to increase the goodness-of-fit, especially in the case of the jump-diffusion models during the crisis period. The jumps have characteristics of a proportional and mean reverting structure. These three main conclusions are supported by all the criteria used. Moreover, they are consistent with the descriptive analysis findings from the previous section, namely the existence of jumps, the non-normality of returns and the non-stationarity of the price process.

5. Conclusion

This study focuses on the behavior of asset returns which capture the excess kurtosis and heavier tails (i.e. leptokurtic behavior). It is well known that asset return distributions are heavy tailed and skewed, which are at odds with the classical geometric Brownian motion models. This excess kurtosis is accounted for by a jump-diffusion model like Merton's and others. In capturing the dynamics of the Korean CDS spread. This study notably focuses on the comparison of mean reverting drifts and jump part of the continuous-time models of CDS spread. The results are as followings. First, empirical findings indicate that the addition of jumps leads to a lower expected return (μ) and volatility (σ). This result implies that jumps account for a substantial portion of the overall volatility of the return data. Second, during pre and post crisis period in Korean CDS market we can observe that the GBM is better than competing models in terms of parameter significance, log-likelihood and the BIC. Third, the addition of jumps improves performance significantly since all jump diffusion processes outperform their diffusion counterparts especially during the crisis period. The findings indicate. CDS prices have jumps caused by large movements and shocks during the crisis period. Finally, the addition of mean-reversion appears to increase the goodness-of-fit, especially in the case of the jump-diffusion models during the crisis period. The jumps have characteristics of a proportional and mean reverting structure. Empirical findings are consistent with the descriptive analysis findings, namely the existence of jumps, the non-normality of returns and the non-stationarity of the price process.

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