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## **MOVEMENT OF INTRA-INDUSTRY TRADE INDEX IN TERMS OF EXCHANGE RATE CHANGE: THEORETICAL ANALYSIS BASED ON A TWO-STAGE PRODUCTION MODEL**

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### **Abstract**

This paper analyzes the movement of intra-industry trade index when the exchange rate or the world price changes. In our two-stage production model, firms construct factories in a foreign country by foreign direct investment (FDI). The firms export components from the home country to the foreign factories, and import final products from the foreign factories. The foreign factories also sell the final products in the world market. Under this knockdown system, we research the movement of the intra-industry trade index of the home country. This paper shows that appreciation of the exchange rate does not always raise the intra-industry trade index. We also show that changes in the world price have the similar effect on the movement of the intra-industry trade index.

**Keywords:** Intra-Industry Trade Index, Foreign Direct Investment (FDI), Exchange Rate

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### **1. Introduction**

In recent years, the intra-industry trade of Japan has been increasing. Several research studies interpret this phenomenon as a sign of change in the industrial structure in Japan and other countries. Watanabe and Kajiwara (1983) calculate the manufactured goods' intra-industry trade index between Japan and foreign countries, and show that the trade pattern between Japan and Asian countries has been approaching the trade pattern between Japan and developed countries, and that the industrialization in Asian countries has been rapidly increasing. Park and Park (1991) analyze the intra-industry trade index for manufactured goods of each factor intensity (raw material, labor, capital, and technology). These articles argue that economic relations between Japan and foreign countries have become strong.

One of the representative Japanese industries, which makes foreign direct investment (FDI) in Asian countries, is the machinery industry. In this industry, production processes can be easily separated. Some processes are transferred to affiliates in foreign countries by FDI (Park and Park, 1991). This means that the increase in FDI has caused growth in the vertical intra-industry trade in which Japan exports components and imports final products.

An important reason for the increase of Japanese FDI in Asian countries is the

appreciation of the Japanese currency. Appreciation of the yen has accelerated Japanese FDI and imports, which has increased vertical intra-industry trade.

In the literature, there are plenty of theoretical research on vertical intra-industry trade. Helpman and Krugman (1985), Kojima (1979a, 1979b), and Dixit and Grossman (1982) contain models of this kind of intra-industry trade. Helpman and Krugman (1985) demonstrate a trade model of the  $H=O=S$  type, in which the capital-abundant country separates the production processes and moves the labor-intensive production process to the labor-abundant country by FDI. The capital-abundant country exports the components and imports the final products. Kojima (1979) describes a trade model of the comparative advantage type. The production process is separated into production of the components and assembly. A country exports either the components or the final products according to comparative advantage, and imports the other. Dixit and Grossman (1982), on the other hand, describe a trade model of the absolute advantage type. The production process of a good is divided into the labor-intensive production process and the capital-intensive production process. A country exports either the components or the final products according to absolute advantage, and imports the other. These three models clearly explain the intra-industry trade of components and final products.

However, these models do not analyze the influence of the appreciation of currency, which affects a firm's revenue and production costs. In addition, in the case for Japanese machinery industry's FDI in Asian countries, the sales share of products exported to Japan is not very high. We must consider the sales to the international market when we construct a vertical intra-industry trade model. Furthermore, there has been little theoretical research about the movement of the intra-industry trade index based on the vertical intra-industry trade model. Consequently, the theoretical research has not analyzed whether appreciation of the currency would raise the index.

We construct a partial equilibrium model with the exchange rate and the international market. This is a vertical intra-industry trade model in which a country exports components and imports final products. The final products are also sold in the world market. In this paper, there are three parts to the analysis. First, we construct the model. Second, we describe how the pattern of production and the amount of trade are decided according to the exchange rate and the world price. Third, we research the movement of the intra-industry trade index. As the index, we use the Grubel=Lloyd index, which is generally used to analyze intra-industry trade. We analyze the movement of the index as the exchange rate or the world price changes.

We prove that the intra-industry trade index does not always increase when the exchange rate appreciates. The relations between appreciation of the exchange rate and the intra-industry trade index are various and complicated. Our theoretical model shows that when the exchange rate appreciates, there are some patterns on the movements of the intra-industry trade index: monotonic increase, monotonic decrease, decrease and increase, and increase and decrease. Furthermore, changes in the world price also have the same effects. Therefore, the increase in the intra-industry trade index does not always indicate the deepening of economic relationship with foreign countries.

The empirical research using the intra-industry trade index has been implemented for a long time. Recently, Ito and Okubo (2016) argue that the conventional intra-industry trade index does not directly address the quality issue and proposes a methodology to make full use of unit price gap information to deduce quality differences between simultaneously exported and imported products. Hayakawa *et al.* (2017) also discuss this issue. Our analysis would contribute to the empirical research using the intra-industry trade index.

This paper is organized as follows. In Section 2, we explain the model. In Section 3, we use the Grubel=Lloyd index and research the movement of the index according to changes in the exchange rate. In Section 4, we research the movement of the index according to changes in the world price. Section 5 presents our conclusion.

## 2. The Model

We consider an industry where firms have a two-stage production process; they make components, then they assemble them. Both stages can be performed in a foreign country. The

firms in each country do not buy components from other firms. The components and the final products are classified as goods of the same industry. We neglect transport costs and tariffs.

We assume that there is a barrier of entry into the home market. In other words, the market of the home country and the world market are separate. The number of firms in the home country is  $n$ . Each oligopolistic firm has the same cost function. The world price of the final product is given. Each oligopolistic firm sells its final products in both the home market and the world market, and behaves to maximize its profit.

The exchange rate is  $e$ ; when the currency of the home country appreciates, the value of  $e$  falls. The quantity of components and final products that a firm makes are  $x$  and  $z$ , respectively. The total cost in each production stage is as follows (Table 1).

**Table 1. The total cost in each production stage**

	The home country	The foreign country
Production of components	$a_{11}x^2 + b_{11}$	$ea_{21}x^2 + b_{11}$
Assembly	$a_{12}z^2 + b_{12}$	$ea_{22}z^2 + b_{12}$

**Note:** The cost of assembly does not include the cost of the production of components.

We assume that the fixed cost,  $b_{11}$  and  $b_{12}$ , is the same in the home country and the foreign country. This is because a firm uses the same capital good in both countries. The coefficients of the valuable cost,  $a_{11}$ ,  $a_{12}$ ,  $a_{21}$  and  $a_{22}$ , are decided by wage, price of material goods, productivity, and other factors.

Furthermore, we assume that each firm does not make the components in both countries at the same time because the fixed cost is very high. The components are produced only in the one place, either in the home country or the foreign country. This assumption is also applied to assembly, that is, the production of the final goods.

Thus, the oligopolistic firms move the location of their factories according to appreciation of the currency.

No. 1.  $e \geq a_{12} / a_{22}$  At this time,

$$a_{11}x^2 + b_{11} < ea_{21}x^2 + b_{11} \quad \text{and} \quad a_{12}z^2 + b_{12} \leq ea_{22}z^2 + b_{12} \quad (2.1)$$

are realized. Production of both the components and the final goods is carried out in the home country.

No. 2.  $a_{11} / a_{21} < e < a_{12} / a_{22}$  At this time,

$$a_{11}x^2 + b_{11} < ea_{21}x^2 + b_{11} \quad \text{and} \quad a_{12}z^2 + b_{12} > ea_{22}z^2 + b_{12} \quad (2.2)$$

are realized. The firms make the components in the home country and assemble them in the foreign country. In other word, they adopt the knockdown system.

No. 3.  $e \leq a_{11} / a_{21}$  At this time,

$$a_{11}x^2 + b_{11} \geq ea_{21}x^2 + b_{11} \quad \text{and} \quad a_{12}z^2 + b_{12} > ea_{22}z^2 + b_{12} \quad (2.3)$$

are realized. Production of both the components and the final goods is carried out in the foreign country.

In the case of No. 2, vertical intra-industry trade of components and final products occurs. Therefore,

$$a_{11} / a_{21} < e < a_{12} / a_{22} \quad (2.4)$$

is the condition under which intra-industry trade occurs. We research the intra-industry trade index for this situation.

### 3. Changes in the Exchange Rate and the Intra-Industry Trade Index

In the following sections, we assume that the exchange rate is that of (2.4) and that the oligopolistic firms adopt the knockdown system, which means that the firms produce the components in the home country and assemble them in the foreign country. Then, they sell some of the final products in the world market, and import the rest into the home country. We also assume that the world price of the final product denominated in the foreign currency,  $p_w$ , is given. In this section, we analyze the movement of the intra-industry trade index as the exchange rate changes under these assumptions.

For simplicity, we assume that one final product is made from *one component*. Therefore, let  $x$  denote the output of components and the output of final products. The total cost in each production stage is as follows (Table 2).

**Table 2. The total cost in each production stage of the knockdown system**

	In the home country	In the foreign country
Production of components	$a_{11}x^2 + b_{11}$	
Assembly		$ea_{22}x^2 + b_{12}$

We assume that each oligopolistic firm exhibits Cournot behavior and decides the sales quantity. The inverse demand function in the home country is

$$p = s - tY, \quad (3.1)$$

where  $p$  is the price of the final product in the home country and  $Y$  is the sales quantity of all firms in the home country.

#### 3.1. Quantity and Price of Trade

The  $i$ th firm's marginal cost of production of the final goods,  $MC_i$ , is

$$MC_i = 2(a_{11} + ea_{22})x_i, \quad (3.2)$$

where  $x_i$  is the output of the final goods of the  $i$ th firm.

By (3.1), the  $i$ th firm's total revenue in the home market is  $py_i = (s - tY)y_i$ , where  $y_i$  is the  $i$ th firm's sales quantity in the home country ( $\sum_{i=1}^n y_i = Y$ ). Therefore, the marginal revenue in the home market,  $MR_i$ , is

$$MR_i = s - t \sum_{j \neq i}^n y_j - 2ty_i. \quad (3.3)$$

We assume that the positions of the marginal cost curve,  $MC_i$ , the marginal revenue curve,  $MR_i$ , and the world price of the final product denominated in the home currency,  $ep_w$ , are as described in Figure 1. The horizontal line,  $ep_w$ , represents the marginal revenue curve in the

world market.

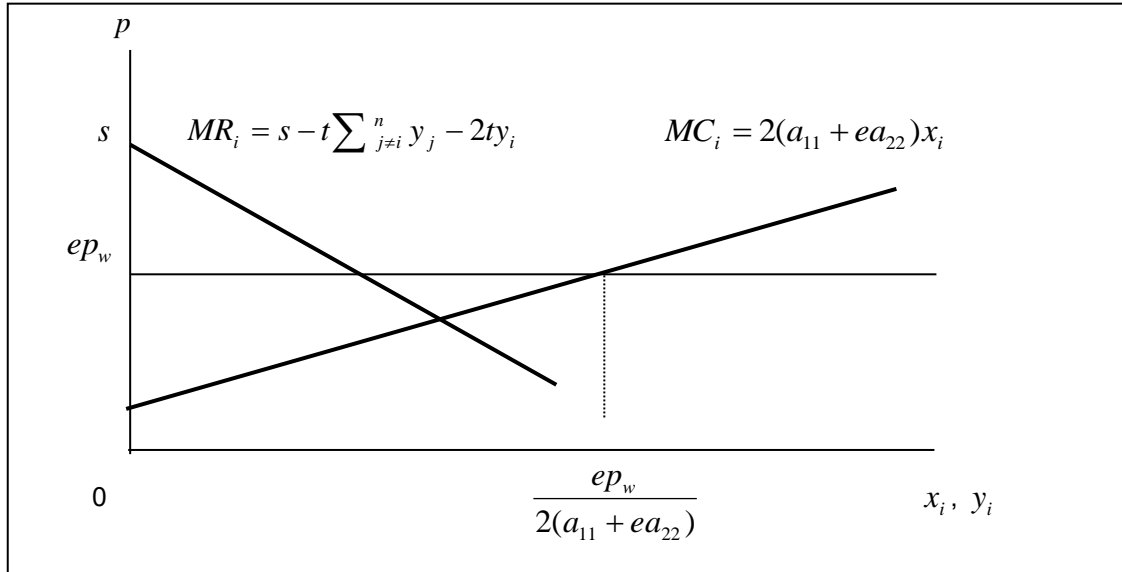


Figure 1. The marginal cost curve and marginal revenue curve of a firm

At this time, the output of the final products of the  $i$  th firm is<sup>1</sup>

$$\frac{ep_w}{2(a_{11} + ea_{22})}. \quad (3.4)$$

This is also the quantity of export of the components.

The sales quantity of the final products in the home country is decided where the marginal revenue is equal to the marginal cost. This condition is expressed as

$$s - t \sum_{j \neq i}^n y_j - 2ty_i = ep_w, \quad j = 1, \dots, n.$$

If we add them with all  $j$ , we obtain the all sales quantity in the home market,

$$Y = \frac{n(s - ep_w)}{(n + 1)t}. \quad (3.5)$$

This value is also the quantity of import of the home country.

A firm's sales quantity in the home market is

$$y = \frac{s - ep_w}{(n + 1)t}.$$

The price of the final product in the home market is

$$p = \frac{s + nep_w}{n + 1}. \quad (3.6)$$

<sup>1</sup> See Krugman and Obstfeld (1991) pp.143-145.

We assume that the price of the components is the same as the marginal cost of their production. The marginal cost of the production of the components is  $2a_{11}x_i$ , so the price of the components denominated in the home currency is

$$\frac{ea_{11}p_w}{a_{11} + ea_{22}}. \quad (3.7)$$

### 3.2. Amount of Trade

The export price of the components denominated in the foreign currency,

$$\frac{a_{11}p_w}{a_{11} + ea_{22}}, \quad (3.8-1)$$

increases as  $e$  falls, whereas the export price of the components denominated in the home currency decreases as  $e$  falls.

The quantity of the components exported from the home country,

$$\frac{nep_w}{2(a_{11} + ea_{22})}, \quad (3.8-2)$$

decreases as  $e$  falls.

The import price of the final product denominated in the foreign currency,

$$\frac{s + nep_w}{(n+1)e}, \quad (3.8-3)$$

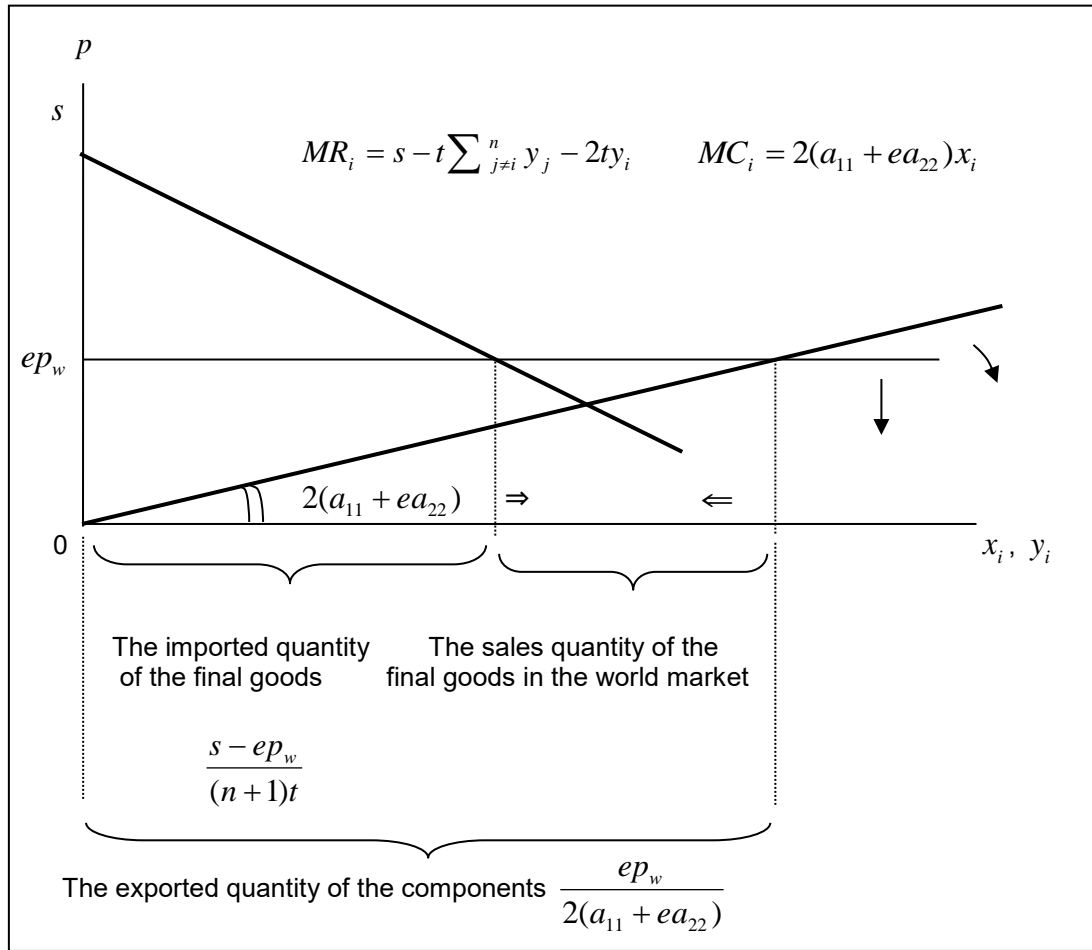
increases as  $e$  falls, whereas the price of the final product denominated in the home currency decreases as  $e$  falls.

The quantity of the final products imported into the home country,

$$\frac{n(s - ep_w)}{(n+1)t}, \quad (3.8-4)$$

increases as  $e$  falls. This is because the price of the final product denominated in the home currency decreases as  $e$  falls.

Appreciation of the exchange rate diminishes the production cost in the foreign country, but it also diminishes the profit from sales in the world market denominated in the home currency. Therefore, the quantity of exported components decreases and that of imported final products increases. These changes in the quantity are explained in Figure 2. The gradient of the marginal cost curve  $MC_i$  is  $2(a_{11} + ea_{22})$ . As  $e$  falls, the curve turns toward the horizon. On the other hand, the horizon line of  $ep_w$  moves downward as  $e$  falls. As a result, the quantity of exported components decreases, and the quantity of imported final products increases. These movements are described with arrows.



**Figure 2. Change in export and import as the exchange rate appreciates**

The amount of export and import is the product of the price multiplied by the quantity. We denote the amount of export denominated in the foreign currency as  $X$ , and the amount of import denominated in the foreign currency as  $M$ , respectively. These are

$$X = \frac{nea_{11}p_w^2}{2(a_{11} + ea_{22})^2}, \quad (3.9-1)$$

$$M = \frac{n(s + nep_w)(s - ep_w)}{(n+1)^2 et}. \quad (3.9-2)$$

According to the above analysis,  $M$  increases as  $e$  falls. On the other hand,  $X$  may not change monotonically as  $e$  falls.

### 3.3. Amount of Trade When the Exchange Rate Is Very High or Very Low

In the above sections, we assume that firms sell the final products both in the world market and in the home market. However, as Figure 1 shows, if  $ep_w$  is larger than  $s$ , that is,

$$e > s / p_w \quad (3.10)$$

is realized, all final products are sold in the world market and no products are imported into the home country.  $M = 0$  is realized.

On the other hand, as Figure 3 shows, if the position of the horizon line  $ep_w$  is lower than the intersection of the curve  $MR_i$  and the curve  $MC_i$ , no final products are sold in the world market and all products are imported into the home country. The marginal cost at the intersection of the curve  $MR_i$  and the curve  $MC_i$  is

$$MC^* = \frac{2s(a_{11} + ea_{22})}{t(n+1) + 2(a_{11} + ea_{22})}. \quad (3.11)$$

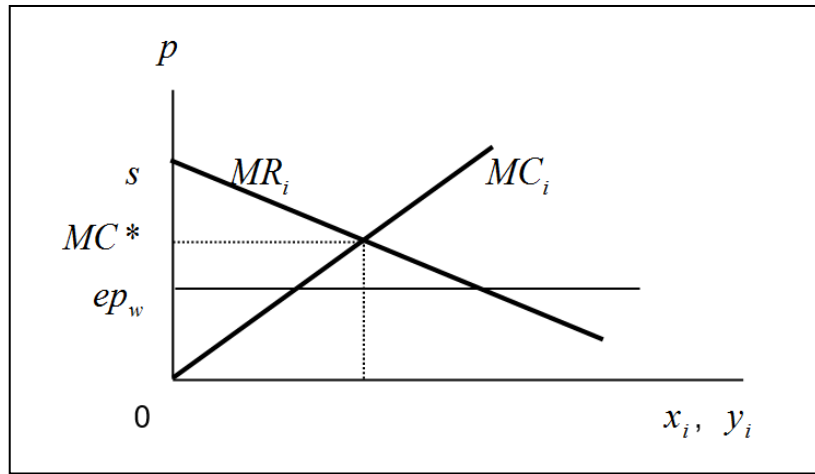


Figure 3. When the world price is very low

All final products are imported when  $e$  is very small so as to realize  $ep_w < MC^*$ . At that time, we obtain

$$X = \frac{ns^2 \cdot 2a_{11}}{e\{(n+1)t + 2(a_{11} + ea_{22})\}^2}, \quad (3.12-1)$$

$$M = \frac{ns^2(2a_{11} + 2ea_{22} + t)}{e\{(n+1)t + 2(a_{11} + ea_{22})\}^2}. \quad (3.12-2)$$

It is obvious that  $X < M$  is realized. This is because the price of the final product is higher than the price of the components.

### 3.4. Locus of the Intra-Industry Trade Index with Changes in the Exchange Rate

We define the intra-industry trade index (IIT index) as



$$IIT = 1 - \frac{|X - M|}{X + M}. \quad (3.13)$$

We assume the world price,  $p_w$ , is given. Therefore, the value of the IIT index is decided by the value of the exchange rate,  $e$ .

No. 1  $e > s/p_w$  At this time,

$$IIT = 0.$$

This is because  $M = 0$ .

No. 2  $e_1 \leq e \leq s/p_w$  ( $e_1$  is the value of  $e$  at which  $ep_w = MC^*$  is realized.) At this time,

$$IIT = 1 - \frac{|X - M|}{X + M},$$

where  $X$  and  $M$  are those of (3.9).

No. 3  $e < e_1$  At this time,

$$IIT = \frac{4a_{11}}{2ea_{22} + 4a_{11} + t}.$$

All final products produced in the foreign country are imported.

Let us now consider the locus of the intra-industry trade index. In No. 1,  $X > M = 0$  is realized. Therefore, in No. 2, there is at least one value of  $e$  where  $X = M$  is realized. If  $X$  decreases monotonically as  $e$  falls, the IIT index increases monotonically and reaches 1 ( $X = M$  is realized), then decreases monotonically. In No. 3, the IIT index increases again as  $e$  falls. These movements are described in Figure 4.

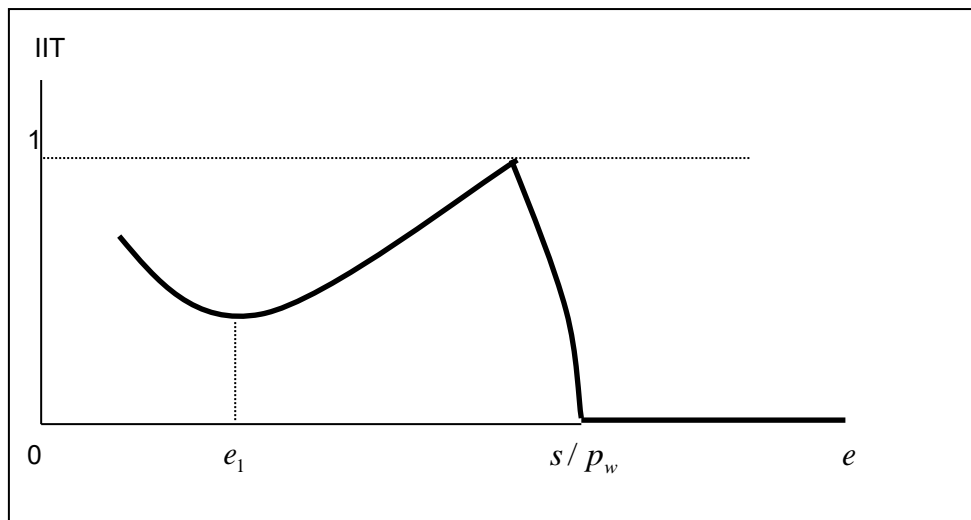
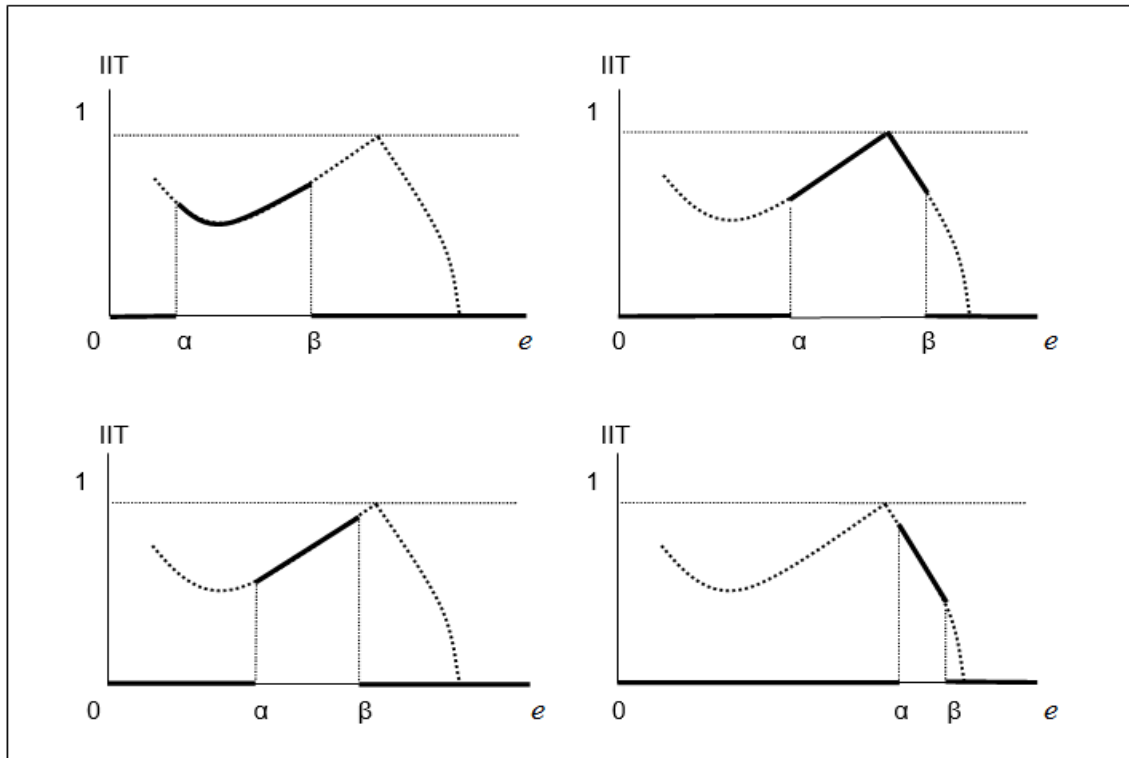


Figure 4. Intra-industry trade index with changes in the exchange rate

However, if  $X$  does not change monotonically as the exchange rate appreciates, the IIT index would move in a more complicated manner.

### 3.5 Restraint of the Range of the Exchange Rate

As we said in Section 2, the necessary condition for the knockdown system is (2.4). We denote  $a_{11}/a_{21} = \alpha$  and  $a_{12}/a_{22} = \beta$ , and the curve described in Figure 4 may be restrained by the values of  $\alpha$  and  $\beta$ . This is because  $IIT = 0$  is realized when  $e \leq \alpha$  or  $e \geq \beta$ . The examples of this restraint are described in Figure 5.

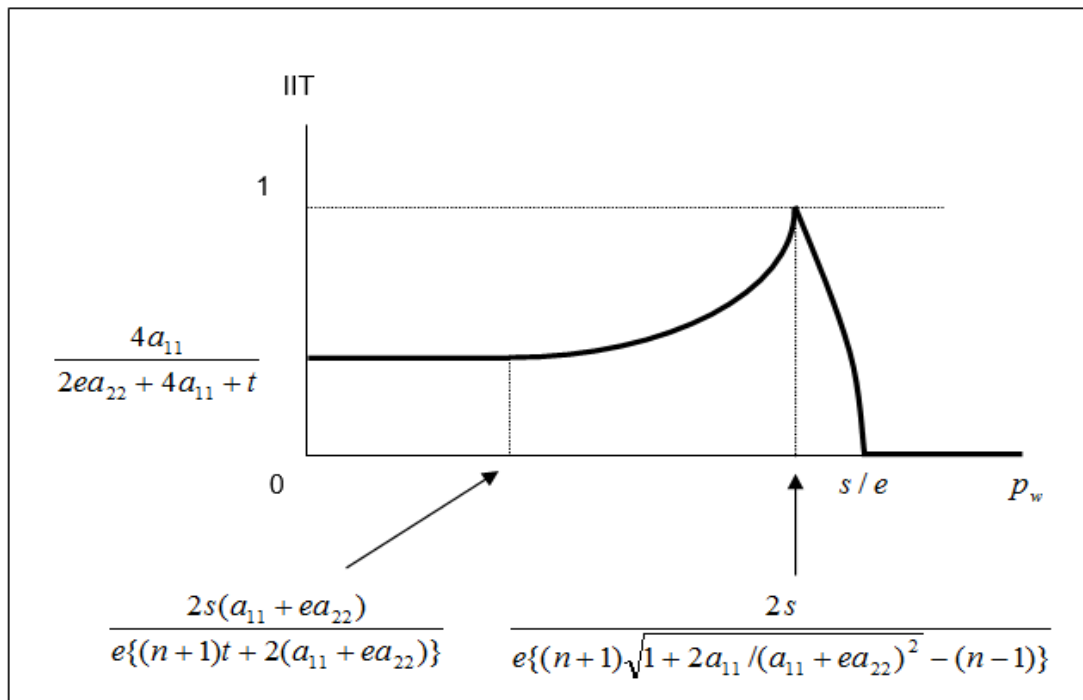


**Figure 5. Intra-industry trade index with changes in the exchange rate**

Figure 5 shows that the IIT index moves in various ways as  $e$  falls: monotonic increase, monotonic decrease, decrease and increase, and increase and decrease.

### 4. Changes in the World Price and the Intra-Industry Trade Index

In this section, we assume that the exchange rate is that of (2.4), that each firm applies the knockdown system, and that the exchange rate,  $e$ , is given. We analyze the movement of the intra-industry trade index as the world price,  $p_w$ , changes. The locus of the intra-industry trade index under this condition is described in Figure 6.



**Figure 6. Intra-industry trade index with changes in the world price**

The meaning of Figure 6 is as follows. As the world price rises, the quantity and price of the final product supplied to the world market increase. On the other hand, as the world price rises, the price of import increases but the quantity of import decreases. Therefore, as the world price rises, the trade deficit of the home country decreases and then the trade surplus increases. Thus, the IIT index curve reaches 1, and then falls.

The shape of the IIT index curve in Figure 6 is simpler than that in Figure 4. When the exchange rate changes, the marginal cost curve,  $MC_i$ , and marginal revenue curve,  $ep_w$ , move. However, when the world price changes, only the marginal revenue curve,  $ep_w$ , moves.

The conclusions in Sections 3 and 4 are valid when the cost function is extended from  $x$  squared to  $x$  to the  $m$ th power ( $m > 1$ ). See Appendix.

## 5. Conclusion

We constructed a model of vertical intra-industry trade in which a country exports components and imports final products. Using this model, we analyzed the movement of the intra-industry trade index. We proved that the intra-industry trade index does not always increase when the exchange rate appreciates. The relations between appreciation of the exchange rate and the intra-industry trade index are various and complicated. When the exchange rate appreciates, there are some patterns on the movements of the intra-industry trade index: monotonic increase, monotonic decrease, decrease and increase, and increase and decrease. Furthermore, changes the world price also have the similar effects on the movements of the intra-industry trade index. Therefore, the increase in the intra-industry trade index does not always indicate the deepening of economic relationship with foreign countries.

Finally, we examine limitations of our analysis. First, we assumed the existence of an oligopolistic market in the home country, and that other firms cannot enter the market. It does sometimes happen that the market of a certain kind of manufacture is oligopolistic. If this assumption does not exist, the entry of a new firm would diminish the gap between the price in the home country and the world price. Second, in contrast to this, if firms have production

functions of increasing returns to scale, the oligopolistic firms may have stronger powers to decide the price in the home market, or the price in the world market. Third, we assumed that a world market for the components does not exist. We could extend our model on this point.

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## Appendix. Generalization of the Cost Function

The cost function of each firm is extended from  $x$  squared to  $x$  to the  $m$ th power ( $m > 1$ ). This means that the decreasing returns to scale remains assumed (Table A1).

**Table A1. The total cost in each production stage on the generalized cost function**

	In the home country	In the foreign country
Production of components	$a_{11}x^m + b_{11}$	
Assembly		$ea_{22}x^m + b_{12}$

The value of the IIT index is decided with the value of the exchange rate,  $e$ .

No. 1'  $e > s/p_w$

$$\text{IIT} = 0.$$

No. 2'  $e_2 \leq e \leq s/p_w$  ( $e_2$  is the value of  $e$  at which  $ep_w = m(a_{11} + ea_{22})x^{m-1}$  is realized. In addition,  $x$  is the solution of  $s - (n+1)tx = m(a_{11} + ea_{22})x^{m-1}$ .)

$$\text{IIT} = 1 - \frac{|X' - M|}{X' + M},$$

$$\text{where } X' = \frac{ne^{1/(m-1)}a_{11}p_w^{m/(m-1)}}{m^{1/(m-1)}(a_{11} + ea_{22})^{m/(m-1)}}, M = \frac{n(s + nep_w)(s - ep_w)}{(n+1)^2 et}.$$

No. 3'  $e < e_2$

$$\text{IIT} = \frac{2ma_{11}x^{m-1}}{ma_{11}x^{m-1} + s - ntx},$$

where  $x$  is the solution of  $s - (n+1)tx = m(a_{11} + ea_{22})x^{m-1}$ .

The shape of the IIT index curves may be more complicated. However, the conclusions of our analysis do not change.