EURASIAN JOURNAL OF SOCIAL SCIENCES

www.eurasianpublications.com

WAVELET ANALYSIS OF THE EURO AND ITS CO-MOVEMENT WITH FOUR EXCHANGE RATES

Meifen Chu
Kyushu University, Japan
Email: chu@econ.kyushu-u.ac.jp

Abstract

The objective of this study is to examine the characteristics of the Euro exchange rate and its co-movement with four main currencies (the AUD, POUND, YEN, and RMB). After the Euro was adopted as an official currency in EU area, it grew rapidly and became the second most-traded currency in the world. The increasing importance of the Euro drew much attention from researchers and policymakers. This paper employs the wavelet analysis, which has become popular and recently is applied in various fields, such as medical science, radio science and social science. First, this paper observes five currency exchange series: the EURO, AUD, POUND, YEN and RMB. Next, using wavelet analysis, it examines the characteristics of each series. Then, a wavelet coherence analysis is used to examine the interdependence between the EURO and the other four currencies. The results indicate that the Euro displays a greater interdependence with the AUD and POUND, while it has lesser correlations with the YEN and RMB. This implies the Euro co-moves with the American and European exchange markets while it is less related with the Asian counterpart. It is interesting to note that the Yen led the Euro during the global financial crisis. Further, the Euro has interdependence with the four exchange rates at shorter periodicities during the crisis periods and is correlated with the four at long periodicities in the long run.

Keywords: Euro Exchange Rate, Nonlinear Time Series Analysis, Co-movement, Interdependence, Wavelet Analysis

1. Introduction

Since the Euro was introduced as a sole official currency in January 1999, it has grown rapidly and become world's second most-traded currency after the US dollar (Chu, 2017). The Euro has become one of the most important currencies for international settlement and central governments' reserves. Thus far, a large number of studies have been conducted on the Euro exchange rate. Many researchers have studied the determinants of the Euro exchange rate fluctuation (De Grauwe, 2000, 2014). De Grauwe and Grimaldi (2002) analyzed the relationships between different fundamentals and the Euro exchange rate. They found that the fundamentals did not impact the fluctuation of Euro exchange rate. In addition, Shams (2005) found that the belief in the Euro is the most important factor that influenced the Euro movement. De Zwart *et al.* (2009) also conducted an analysis on the Euro movement. However, they concluded that both the fundamentals and technical factors were important in predicting the Euro exchange rate. Moreover, they found that short-term movements were related to technical factors.

On the other hand, some studies focus on analyzing the interdependence of the Euro and other currencies. Trencaa *et al.* (2015) applied the GARCH model, which was proposed by Engle (1982, 1990), to analyze the interdependence between the Euro and USD, Euro and Pound, Euro and CHF, Euro and Yen. They found that structural breakpoints are present. Glick and Rose (1999) examined the volatility spillover between the Euro and Pound and found a contagion relationship between these two currencies. Kitamura (2007) investigated the interdependence and volatility spillover among the Euro, Pound and Swiss franc.

There is a clear lack of dynamic studies on Euro exchange rate and its interdependence with other major exchange rates over different timescales. In an alternative approach, this paper applies wavelet analysis to detect the characteristics of the Euro exchange rate and its dynamic interdependence maps with the AUD, POUND, YEN and RMB, which can precisely examine the factors influencing the fluctuation of the Euro exchange rate.

The contributions of this study are as follows: First, this paper applies wavelet analysis to examine the dynamic characteristics of the Euro exchange rate and its dynamic interdependence with four major exchange rates. This style of analysis is a nonlinear approach widely applied in various fields. However, it has not been commonly employed in studying the Euro exchange rate. Second, the sample period includes the 2008 global financial crisis, the European debt crisis, and Brexit, which are considered the most significant factors impacting the Euro exchange rate. Third, the empirical analysis is novel in the context of checking long-term coherence between the Euro and these four currencies. These analytical processes can be easily applied by future policymakers and investors.

The rest of this paper is organized as follows. Section 2 introduces the wavelet analysis. In section 3, the data set is described. The empirical analysis is applied in section 4. The conclusion as well as the directions for future research are depicted in the closing section.

2. Wavelet transform analysis

Wavelet transform theory was first proposed by two French engineers Grossman and Morlet (Morlet *et al.* 1982a, 1982b; Grossmann and Morlet, 1985; Everson and Sirovich, 1990). This theory became popular in the field of signal processing because it overcame the limitations of the Fourier transform which cannot capture time information. Wavelet transform theory can successfully detect localized information in both time and frequency (Morlet *et al.* 1982a, 1982b; Mallat, 1998, 1999, 2008; Mandelbrot, 1982, 1999). Recently, the theory has been extended so that it can be applied to the fields of engineering, medical science, and economics. It is widely applied to examine the correlation between two time series. For example, Jeong *et al.* (2016) employed wavelet analysis to detect Parkinson's disease and Alzheimer's disease. Karabulut *et al.* (2020) applied wavelet analysis to examine the relationship between commodity price and world uncertainty. Demir *et al.* (2020) applied wavelet analysis to identify the relationships between cryptocurrencies and COVID-19. They found a negative relationship between Bitcoin and the number of reported COVID-19 infections and deaths.

Wavelet transform analysis can be introduced as follows (Mallat, 1998, 1999, 2008; Torrence and Compo, 1988; Aguiar-Conraria and Soares (2011; Chu, 2007).

(1) Continuous Wavelet Transform (CWT): Continuous Wavelet Transform is defined by the following formula:

$$Wf(u,s) = \int_{-\infty}^{\infty} \emptyset^{*}\left(\frac{t-u}{s}\right) f(t)dt \tag{1}$$

s denotes scale and u represents time shift. $\emptyset(t)$ is a mother wavelet. $\emptyset(\frac{t-u}{s})$ is derived from the mother wavelet and called as a daughter wavelet and * represents the complex conjugation. $f(t)$ is a function which can be transformed into $Wf(u,s)$ by multiplying a wavelet function. $Wf(u,s)$ is called as the wavelet coefficient, which is a function of a and b. When scale a is shorter (or longer), it means the frequency is higher (or lower). Normally, the mother wavelet should meet the following condition:

$$\int_{-\infty}^{\infty} \emptyset(t)\, dt = 0 \tag{2}$$

(2) Wavelet power spectrum (WPS): The local oscillation power of a time series $f(t)$ is expressed by the wavelet power spectrum (WPS), which is defined as follows:

$$(WPS)_t(s,u) = |W_t(s,u)|^2 \tag{3}$$

WPS can be plotted as scalogram, which gives a visual representation of a wavelet transform, showing the magnitude in a designed color. If the magnitude of the color is strong, it indicates high oscillation. If the magnitude of the color is weak, it means the oscillation is weak.

One of the important applications of wavelet transform is to examine the interdependence of two-time series by wavelet coherence. Wavelet coherence R_{xy} of the time series $x(t)$ and time series $y(t)$ is defined below:

$$R_{xy} = \frac{|S(W_{xy}(s,u))|}{[S(|W_x(s,u)|^2)\, S(|W_y(s,u)|^2)]^{1/2}} \tag{4}$$

where, S is a smoothing operator. R_{xy}^2 is between zero and one. The closer the value of R_{xy}^2 is to 1, the stronger the interdependence between the two-time series $x(t)$ and $y(t)$. Further, phase-difference can be applied to examine interdependent patterns and lead-lag relationships between the different two time series. The phase-difference is defined as follows:

$$\varphi_{xy} = tan^{-1}\left(\frac{Im\{S(W_{xy}(s,u))\}}{Re\{S(W_{xy}(s,u))\}}\right), \quad \varphi_{xy} \in |-\pi, \pi| \tag{5}$$

Regarding different values of φ_{xy}, the two-time series display different interdependent patterns as shown in Table 1.

Table 1. Phase patterns

φ_{xy}	$(-\pi, -\pi/2)$	$(-\pi/2, 0)$	$(0, \pi/2)$	$(\pi/2, \pi)$
Phase Pattern	Out of Phase $X \xrightarrow{leads} Y$	In Phase $Y \xrightarrow{leads} X$	In Phase $X \xrightarrow{leads} Y$	Out of Phase $Y \xrightarrow{leads} X$

3. Data

In this paper, the daily closing data for the EURO, AUD, POUND, YEN and RMB are used. All the exchange rates are expressed in US dollars. The time period is from 1999.01.04 to 2019.12.31 (year, month, day). Data were extracted from the database provided by Investment Com.

Figure 1 shows the time series of the EURO, POUND, AUD, YEN, and RMB exchange rates. Taken as a whole, it is clear that the EURO, POUND, and AUD exchange rates move very differently when compared to the YEN and RMB counterparts.

The EURO, POUND, and AUD display similar movement from 1999 to 2008. The YEN moves differently from the three exchange rates and is distinct from RMB. At the beginning of the EURO's establishment, the EURO, POUND and AUD dropped largely because of the adoption of the new currency – the EURO. The declining trend lasted for more than one year. The three exchange rates then dramatically went up and became stronger. However, this persistent upward tendency did last long due to the global financial crisis which occurred in 2007. They recovered after 2010 but dropped again in 2013. Following a decline of a few years, they did not return to the pre-2007 levels and have faced a long-term depression. These large fluctuations were caused by the European Sovereign Debt Crisis. As an aftermath, this crisis damaged the EURO dramatically.

Interestingly, the YEN did not move in similar ways as the above three exchange rates. In 1999, the YEN soared for a few months and then dropped. After a few years' fluctuation, it began to soar again, and maintained a similar course even during the global financial crisis. After

2013, like the earlier three exchange rates, it began to drop down sharply and then stayed at a lower level for a long time.

Further, the RMB moved independently of the other four exchange rates. There were almost no changes before 2007. Then, it rose until 2014. After a short decline, it rose again and maintained a comparatively strong upwards tendency
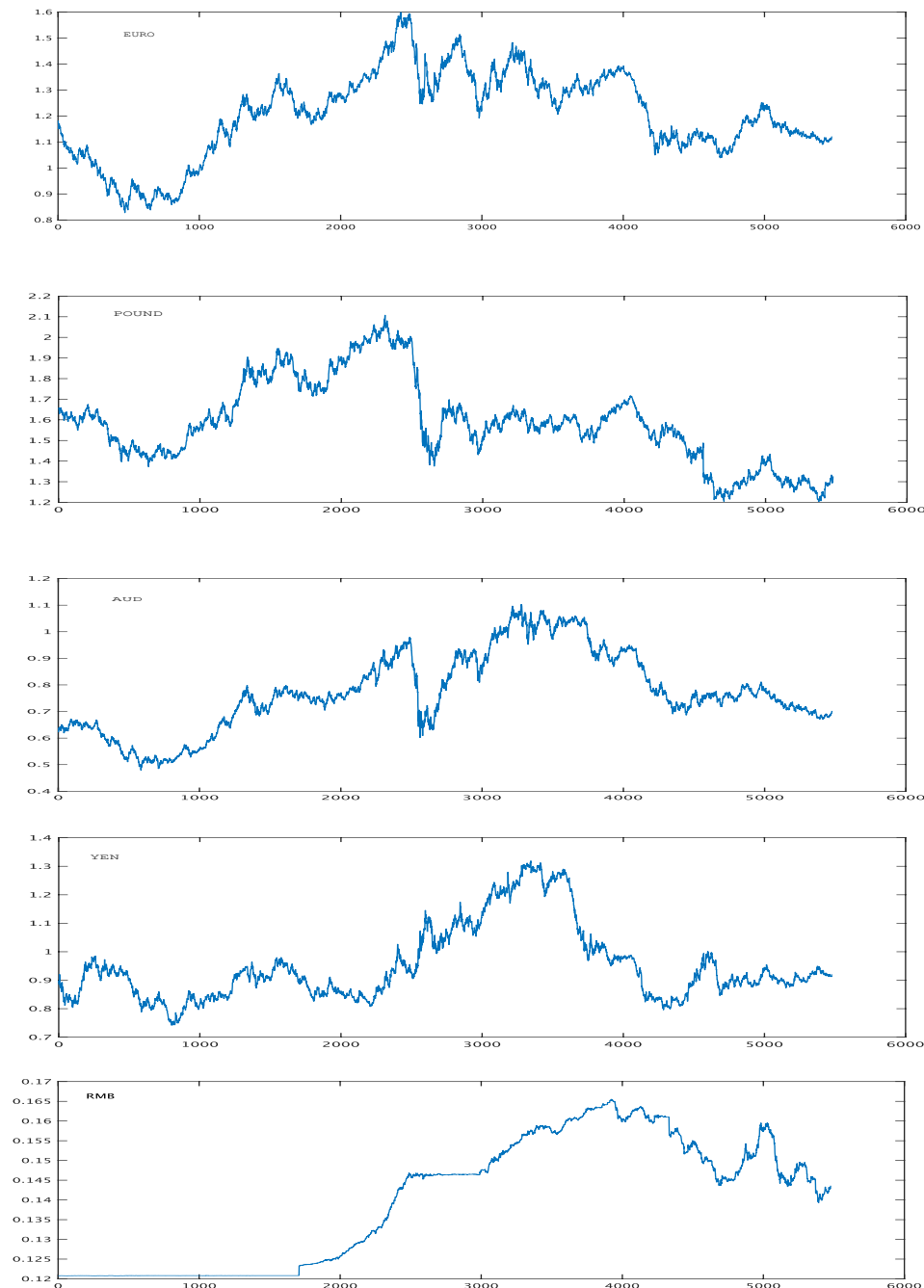

Figure 1. Time series of five exchange rates
Note: From top to bottom, the figures represent the time series of Euro, Pound, AUD, YEN, RMB, respectively. The data period is from 04 Jan 1999 to 31 Dec 2019.
Source: Investment.com (2020)

4. Empirical results
4.1. Wavelet transform analysis results

Figures 2 to 6 plot the wavelet transforms of the EURO, POUND, AUD, YEN and RMB. As shown in Figure 2, the EURO features two significant periodicities. The first is centered around a periodical band of 417 days, from July 2007 to December 2010. The second is centered around a long periodicity of 5 years, from the beginning of the EURO's establishment until August 2008. These results indicate that the Euro's long-term periodical fluctuation started to weaken after 2008. Further, during the global financial crisis and European sovereign debt crisis, the EURO was vibrated in a major manner.
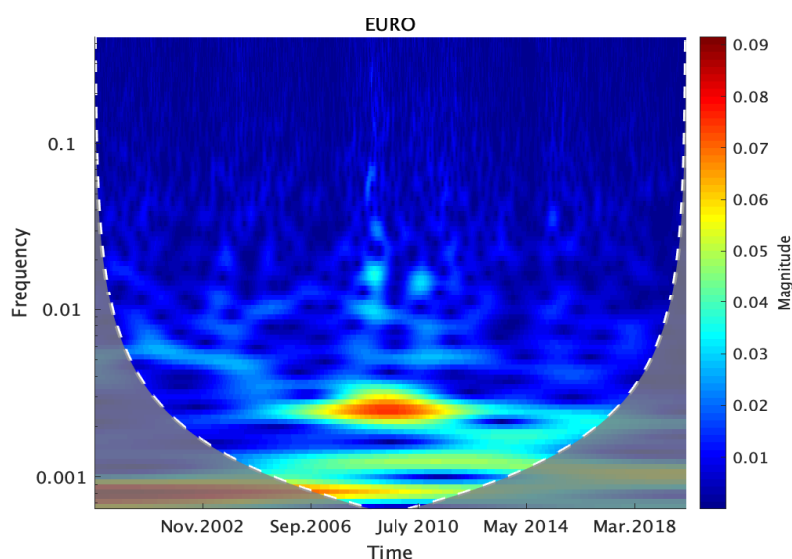
Figure 2. Wavelet Transform of EURO

The AUD experienced large fluctuations in three periodicity bands (as seen in Figure 3). The first is centered around the band of 410 days, from July 2008 to May 2011. The second is centered around 835 days, from June 2005 to July 2011. The third band is centered around 20 years, from April 1999 to May 2010. The AUD fluctuated intensely both at the medium and long periodicity band. This means that the AUD may be easily impacted by other currency markets.
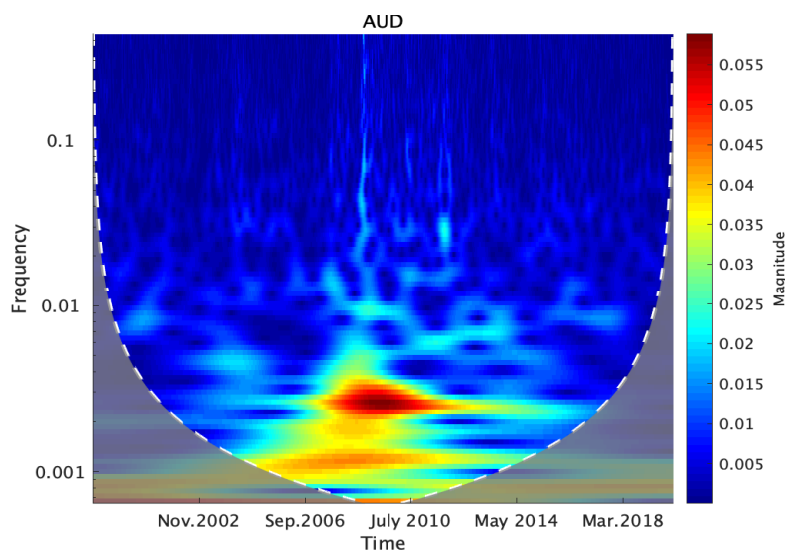

Figure 3. Wavelet Transform of AUD

127

As seen in Figure 4, the POUND displays a strong periodicity band during the period from June 2007 to July 2011 as well. It appears that the EURO, AUD, and POUND fluctuate strongly from July 2007 to July 2010. In other words, the EURO, AUD, and POUND were highly correlated with the US exchange rate market. They were largely affected by the global financial crisis and European Debt Crisis.

In addition, the POUND reveals another strong periodicity band from 1999 to 2013, which is centered around 835 days, from January 2005 to September 2013. A similar strong band is noticed in the AUD but not in the EURO exchange rates. This suggests that the POUND and the AUD were more closely related to the USD exchange market and global markets.
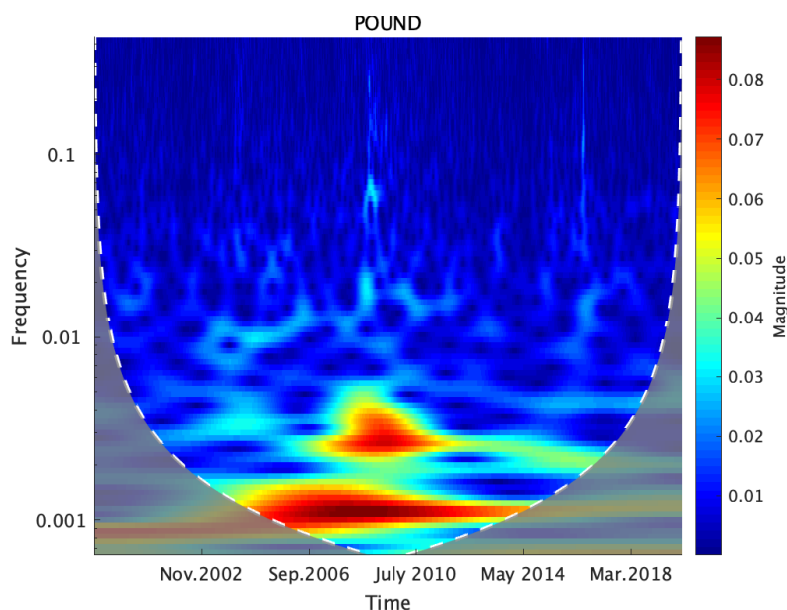
Figure 4. Wavelet Transform of POUND

In contrast, as seen in Figures 5 and 6, both the YEN and RMB reveal no distinct periodicity bands during the whole period.
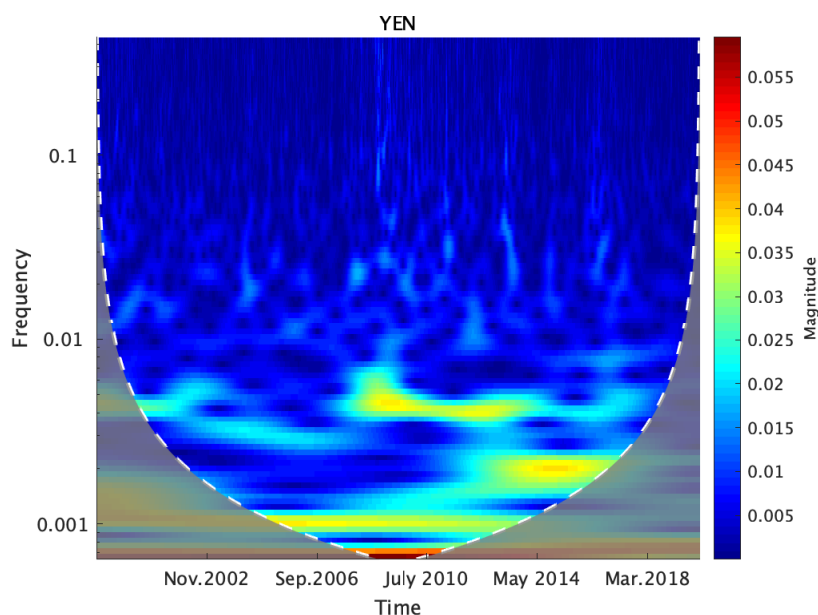
Figure 5. Wavelet Transform of YEN

128

As shown in Figure 5, the YEN displays a long-term periodicity of 20 years. Further, the YEN does show three comparatively weak periodicity bands during the 2008 global financial crisis and the sovereign debt crisis. The first is centered around 223 days, from May 2008 to February 2013. The second is centered around 513 days, from July 2013 to January 2016. The third is centered around 1000 days, form March 2006 to July 2012.

As shown in Figure 6, the RMB displays no significant periodicity bands. Together with Figure 5, this indicates that the YEN and RMB are relatively stable, and they are unlikely to be easily affected by the other markets. In particular, the RMB shows only small fluctuations during the two crises. However, the YEN was impacted by the global crisis and the European sovereign debt crisis. This implies that the two currencies in Asian exchange market are comparatively stable. However, the YEN is globalized rather than the RMB.
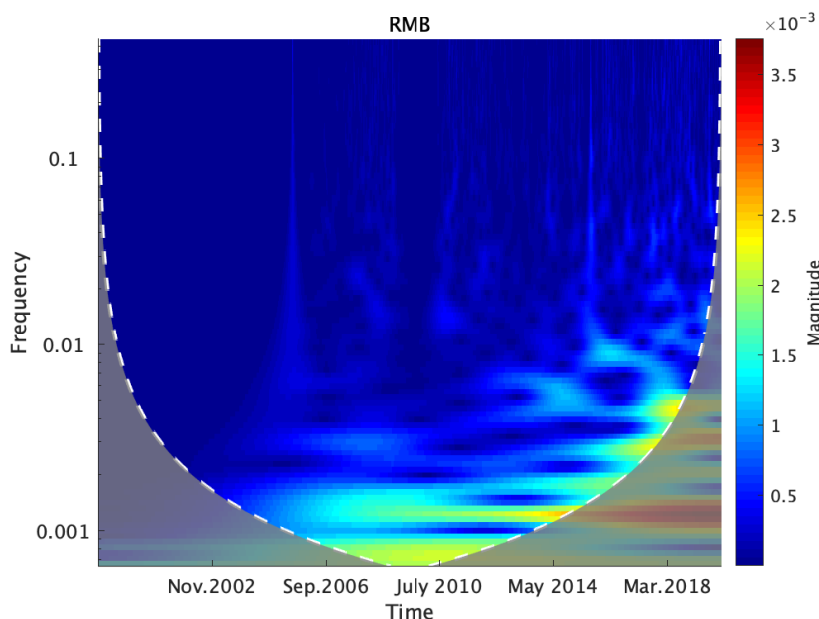

Figure 6. Wavelet Transform of RMB

4.2. Wavelet coherence between EURO and POUND, EURO and AUD, EURO and YEN, EURO and RMB

Wavelet coherence analysis is employed to clarify Euro's interdependencies with four major currencies – AUD, POUND, YEN, and RMB (see Figures 7 to 10). As seen in Figure 7, the EURO is highly correlated with AUD at three periodicity bands. One is centered around 128 days, from November 2002 to April 2011. The second one is centered around 256 days, from September 2001 to May 2014. The third one is centered around 768 days, from August 2003 to August 2008. As seen in Figure 7, in the significant areas, most arrows are pointing from left to right, which means that the coherence between EURO and AUD is in-phase. The EURO reveals a strong correlation with the AUD both at short- and long-term periodicity. In particular, the high interdependence between the EURO and AUD is shown in the periods of the global financial crisis, the European debt crisis and Brexit.
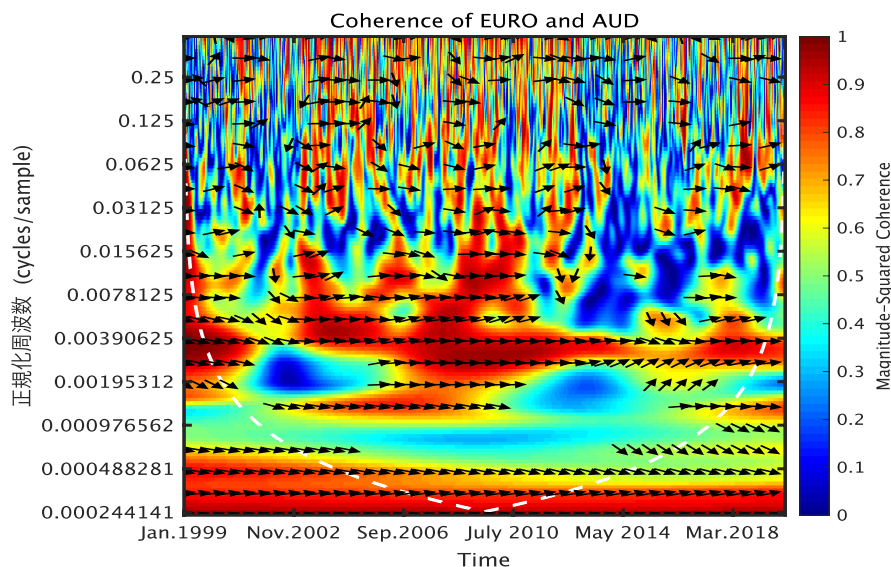
Figure 7. Coherence of EURO and AUD

Figure 8 reveals that that the EURO is significantly coherent with the POUND both at the lower and the upper periodicity bands. Three distinct periodicity bands can be seen. One periodicity band is centered around 96 days from January 1999 to August 2008. Another one is centered around 384 days, from August 2003 to May 2016. The third one is centered around 768 days, from October 2004 to February 2010. This means that the EURO and POUND are closely correlated with each other. Further, the phase patterns between EURO and POUND at different scales are basically displaying in-phase patterns. Interestingly, during the global financial crisis, the arrows point upward, which shows that the EURO was leading POUND. The strong correlation between the EURO and POUND also indicates that the EURO and POUND were greatly impacted by the three large events: the global financial crisis, the European debt crisis, and the Brexit.
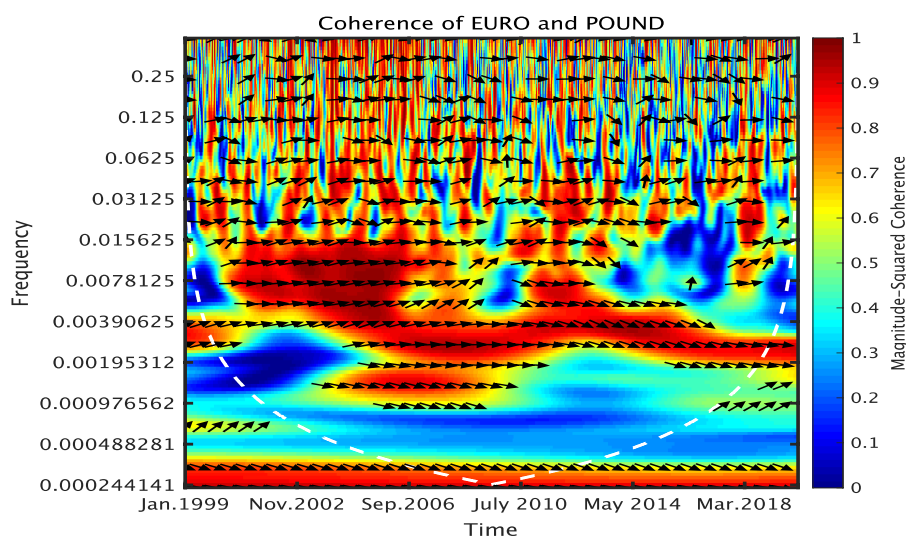

Figure 8. Coherence of EURO and POUND

As seen in Figures 9 and 10, the EURO, on the other hand, displays a weak correlation with YEN and RMB. Specifically, no significant coherence between the EURO and RMB can be seen. Between the EURO and YEN, two periodicity bands can be observed. One is centered around 192 days, from November 2003 to January 2006. The other is centered around 512 days,

from February 2015 to October 2017. According to these phase patterns, it can be seen that the YEN was leading EURO and the EURO was leading RMB during the global financial crisis period. Further, the event of Brexit did not impact the Asian exchange markets.
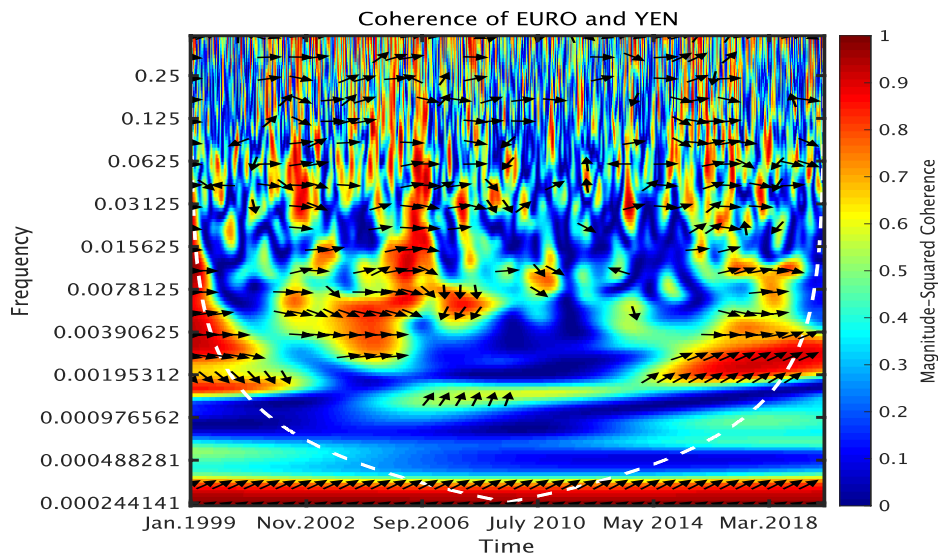
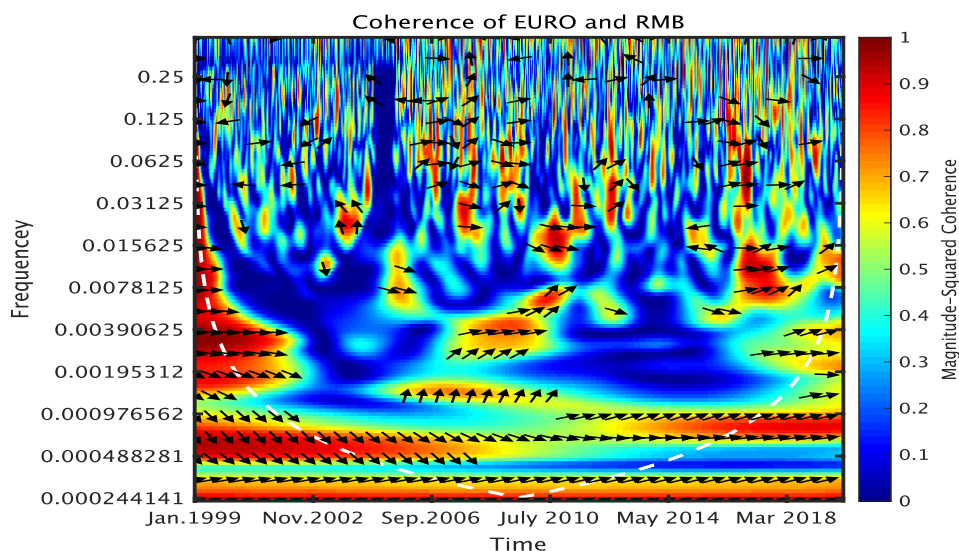

Figure 9. Coherence of EURO and YEN

Figure 10. Coherence of EURO and RMB

5. Conclusion

This study offers an in-depth analysis of the EURO and its co-movement with four major currencies by using a nonlinear approach – wavelet analysis. Wavelet analysis has rarely been applied to study exchange rates, although it has been widely used in other fields such as medical science and engineering.

The results of this paper show that the EURO displays strong periodical vibration both in the short and long periodicity bands. At a long-term (ten years) periodicity, the EURO displays in-phase relationships with the four currencies: POUND, AUD, YEN, and RMB. This implies that the EURO is correlated with the four major exchange rate markets. During the global financial crisis and the European debt crisis, EURO displayed a strong coherence with any of the three exchange

rates (AUD, POUND and YEN). Moreover, a high degree of correlation between the EURO and POUND at all timescales and between the EURO and AUD at long timescales have been observed. At the same time, there is a low degree of correlation between EURO and YEN, and between the EURO and RMB. These results indicate that the EURO has a close interdependence with American market and a distant one with Asian market.

On the other hand, the phase patterns present high correlations between the EURO and POUND, and between the EURO and AUD, which implies the three exchange rates are moving in the same direction. During the 2008 global financial crisis, the EURO leads POUND and RMB, nevertheless, interestingly, the YEN leads EURO during the same crisis period. It implies that EURO has a comparatively less interdependent relationship with the YEN. However, during the global crisis period, the YEN became more significant, and it led the movement of EURO.

Overall, the above results imply that, the Euro, as an international currency, co-moves significantly with the American and European exchange markets both in the short and long run. It is also comparatively less correlated with the Asian exchange markets. However, during the global crisis, the Euro relied more on a comparatively stable currency - the YEN. These findings will be helpful for policymakers and foreign exchange market investors.

In our further research, other risk factors that impact on the Euro exchange rate will be investigated in order to achieve an in-depth analysis of the Euro exchange rate movement.

References

Aguiar-Conraria, L., Soares, M. J., 2011. Business cycle synchronization and the Euro: A wavelet analysis. *Journal of Macroeconomics*, 33(3), pp. 477-489. https://doi.org/10.1016/j.jmacro.2011.02.005

Chu, M., 2007. The analysis of the multifractal processes of stock prices in an artificial market with wavelet transform and its 'application to the analysis of agents' actions. *The Annual Report of Economic Science*, 45, pp. 121-127.

Chu, M., 2017. A nonlinear analysis on the Euro exchange rate using MF-DFA. *Society of Political Economy*, 84, pp. 45-57.

De Grauwe, P. and Grimaldi, M. B., 2002. *The exchange rate and its fundamentals. A chaotic perspective*. CESifo Working Paper No. 639 (6).

De Grauwe, P., 2000. Monetary policies in the presence of asymmetries. *Journal of Common Market Studies,* 38(4), pp. 593–612. https://doi.org/10.1111/1468-5965.00255

De Grauwe, P., 2014. *Exchange rates and global financial policies.* Singapore: World Scientific. https://doi.org/10.1142/8832

De Zwart, G., Markwat, T., Swinkels, L., and van Dijk, D., 2009. The economic value of fundamental and technical information in emerging currency markets. *Journal of International Money and Finance,* 28, pp. 581-604. https://doi.org/10.1016/j.jimonfin.2009.01.004

Demir, E., Bilgin, M. H., Karabulut, G., and Doker, A. C., 2020.The relationship between cryptocurrencies and COVID-19 pandemic. *Eurasian Economic Review,* 10(3), pp. 349-360. https://doi.org/10.1007/s40822-020-00154-1

Engle, R. F., 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of the United Kingdom inflation. *Econometrica*, 50, pp. 987-1007. https://doi.org/10.2307/1912773

Engle, R. F., 1990. The use of ARCH/GARCH models in applied econometrics. *Journal of Economic Perspectives*, 15(4), pp. 157-168. https://doi.org/10.1257/jep.15.4.157

Everson, R. and Sirovich, L., Sreenivasan, K. R., 1990. Wavelet analysis of the turbulent jet. *Physics Letters A*, 145(6- 7), pp. 314-322. https://doi.org/10.1016/0375-9601(90)90941-G

Glick, R. and Rose, A., 1999. Contagion and trade: Why are currency crises regional? J*ournal of International Money and Finance*, 18(1999), pp. 603-617. https://doi.org/10.1016/S0261-5606(99)00023-6

Grossmann, A. and Morlet, J., 1985. Decomposition of functions into wavelets of constant shape, and related transforms. *Mathematics and Physics - Lectures on Recent Results*, pp. 135-165. https://doi.org/10.1142/9789814415125_0004

Investment.com, 2020. *Currencies.* [online]. Available at: <https://www.investing.com/currencies/> [Accessed on 10 June 2020].

Jeong, D. H., Kim, Y. D., Song, I. U., Chung, Y. A., and Jeong, J., 2016. Wavelet energy and wavelet coherence as EEG biomarkers for the diagnosis of Parkinson's disease-related dementia and Alzheimer's disease, *Entropy*, 18(1), pp. 1-17. https://doi.org/10.3390/e18010008

Karabulut, G., Bilgin, M. H., and Doker, A. C., 2020. The relationship between commodity prices and world trade uncertainty. *Economic Analysis and Policy*, 66, pp. 276-281. https://doi.org/10.1016/j.eap.2020.05.001

Kitamura, Y., 2007. *Interdependence and volatility spillover among the euro, pound sterling and Swiss franc.* Working Paper No. 226.

Mallat, S., 1998. *A wavelet tour of signal processing.* San Diego: Academic Press. https://doi.org/10.1016/B978-012466606-1/50008-8

Mallat, S., 1999. *A wavelet tour of signal processing.* 2nd ed. San Diego: Academic Press. https://doi.org/10.1016/B978-012466606-1/50008-8

Mallat, S., 2008. *A wavelet tour of signal processing.* 3rd ed. San Diego: Academic Press.

Mandelbrot, B., 1982. *The fractal geometry of nature.* San Francisco: Freeman.

Mandelbrot, B., 1999. A multifractal walk down Wall Street. *Scientific American*, 198(2), pp. 70-73. https://doi.org/10.1038/scientificamerican0299-70

Morlet, J., Arens, G., Fourgeau, E., and Giard, D., 1982a. Wave propagation and sampling theory – Part I: Complex signal and scattering in multilayered media: *Geophysics*, 47, pp. 203-221. https://doi.org/10.1190/1.1441328

Morlet, J., Arens, G., Fourgeau, E., and Giard, D., 1982b. Wave propagation and sampling theory – Part II: Sampling theory and complex waves: *Geophysics*, 47, pp. 222-236. https://doi.org/10.1190/1.1441329

Shams, R., 2005. *Dollar-Euro exchange rate 1999-2004 - Dollar and Euro as international currencies.* HWWA Discussion Paper No. 321. https://doi.org/10.2139/ssrn.743064

Torrence, C. and Compo, G. P., 1998. A practical guide to wavelet analysis. *Bulletin of the American Meteorological Society,* 79(1), pp. p.61-78. https://doi.org/10.1175/1520-0477(1998)079<0061:APGTWA>2.0.CO;2

Trencaa, L., Zapodeanub, D., and Cociubaa, M. L., 2015. Testing the presence of structural break in the Euro exchange rate. *Procedia Economics and Finance*, 32(2015), pp. 1163 -1169. https://doi.org/10.1016/S2212-5671(15)01582-8