Eurasian Journal of Business and Management, 12(1), 2024, 1-14
DOI: 10.15604/ejbm.2024.12.01.001

EURASIAN JOURNAL OF BUSINESS AND MANAGEMENT

www.eurasianpublications.com

GENERALIZATION OF LINEAR MODELS FOR MULTIPRODUCT BREAK-EVEN ANALYSIS WITH CONSTANT RATIOS

Sejfudin Zahirović
Faculty of Economics, University of Tuzla, Bosnia and Herzegovina
Email: sejfudin.zahirovic@untz.ba

Jasmina Okičić
Faculty of Economics, University of Tuzla, Bosnia and Herzegovina
Email: jasmina.okicic@untz.ba

Amra Gadžo
Faculty of Economics, University of Tuzla, Bosnia and Herzegovina
Email: amra.gadzo@untz.ba

Received: January 15, 2024 Accepted: April 12, 2024

Abstract

This paper aims to generalize linear models for the multiproduct break-even point. Taking into consideration identified research gaps, the paper focuses on deriving formulas for determining the multiproduct break-even point through determination models. Different assumptions regarding the constancy of individual product contribution structures to total physical production volume, total revenue, total variable costs, and total contribution margin are taken into account. Additionally, connections between the obtained solutions from different models and different assumptions regarding the constancy of individual product contributions are established. The verification of the optimality of solutions obtained through different determination models is conducted by comparing them with solutions derived from linear programming as a benchmark. The developed models are tested using a case study of a multiproduct company in the metal processing industry. Through comparative analysis, the hypotheses concerning obtaining an optimal solution and the identical nature of solutions derived from the determination model and linear programming are examined. This paper contributes to the understanding of the multiproduct break-even point, providing a theoretical and practical framework for evaluation and enabling the application of various determination models in the context of a multiproduct situation.

Keywords: CVP Analysis, Break-even Point, Multiproduct Situation, Linear Programming, Optimization

1. Introduction

For effective business management, managers need to understand the relationships between production volume, costs, revenue, and business results generated at a given production capacity. This analysis is known as Cost-Volume-Profit (CVP) analysis. The break-even point, where the revenue covers the total operating costs at a specific activity level, is often graphically represented as the intersection point of total revenue and total cost functions, referred to as the "break-even point" (BEP). The key outcome of the break-even point analysis is determining the production/sales volume of individual products at which the total revenue equals the total operating costs. This analysis shows us the profitability threshold of the company. However, when interpreting the break-even point, we must consider the applied cost accounting system.
CVP analysis and the break-even point are interconnected concepts. CVP analysis is used to analyze the relationship between prices, sales volume, costs, and profits of a company. The break-even point refers to the sales level at which the revenues cover both variable and fixed costs of the company. The break-even point is a critical concept in CVP analysis as it represents the sales volume at which the revenues equal the variable and fixed costs of the company. This means that the company neither makes a profit nor incurs a loss at that sales quantity. The break-even point can be used as a fundamental indicator for assessing the profitability of the company's operations since the company must sell at least that many products to cover its costs.

The significance of using CVP and break-even point analysis for planning and decision-making in achieving profitability in practical business operations is evidenced by numerous studies (Ilie and Ileana-Sorina, 2017; Lulaj and Iseni, 2018; Stoenoiu, 2018; Le *et al.* 2020; Manjunatha and Raini, 2022). On the other hand, some research has shown that CVP analysis is not adequate for-profit planning in individual business conditions (Wijayanti and Prasetyo, 2021) or has raised numerous criticisms regarding its practical value due to various underlying assumptions (Marjanovic *et al.* 2013).

The paper presents relational equations and corresponding linear programming models developed to determine the multiproduct break-even point with constant ratios of physical production volume, total revenue, variable costs, and individual product margins. Key finding is that all developed models of relational equations and linear programming models provide identical solutions, allowing any of the developed models to be used in the calculation process depending on the available data.

In that respect, the authors selected the subject of this paper on the grounds that not many relevant publications with a focus on the generalization of the linear models for multiproduct break-even analysis can be found in contemporary literature. Based on the identified research gaps, the main objective of this study is to generalize the linear models for multiproduct break-even analysis. To achieve this objective, the following tasks need to be carried out. Derive formulas for determining the multiproduct break-even point using break-even analysis models (conditional product model, average marginal contribution model, system of linear equations model) under different assumptions regarding the constancy of the contribution structure of individual products in relation to total physical output, total revenue, total variable costs, and total contribution margin. Establish the relationships between the obtained solutions using different break-even analysis models and different assumptions regarding the constancy of the contribution ratios of individual products. Verify the optimality of the solutions obtained from different break-even analysis models, with linear programming solutions serving as the benchmark. Test the developed models on a case study of a multiproduct company in the metal processing industry.

The paper is organized as follows. After the introduction, a brief overview of the literature that is applicable to the research is provided. The paper goes on to explain the methodology, after which the findings are discussed. Ultimately, in line with the results of the study, a brief description of the key conclusions is given.

2. Literature review and hypothesis development

The original model of the CVP analysis was first presented by Hess (1903) and Maan (1903-07) (Stefan *et al*. 2008; Dash, 2019) and was based on a single-product model with assumptions of no uncertainty and linearity of cost and revenue functions. However, ideas for this approach appeared earlier in the "general equilibrium theory" of Léon Walras (Ekelund and Hébert, 2014), the concept of "minimum production costs" of William Petty (Roncaglia, 2005), and the concept of "short- run cost coverage" (Hunt and Lautzenheiser, 2011). Various CVP approaches were then developed for a multi-product situation characterized by the production of multiple products that differ from each other in terms of physical units of measure, selling prices, level of demand and unit variable production costs (Johnson and Simik, 1971; Atkinskon and Kaplan, 2007; Stevanović and Petrović, 2016).

CVP analysis traditionally relies on the assumption that cost, and revenue functions are linear for a given range of activities observed in the short-term time interval. The justification for the application of a linear model is advocated by numerous authors, explaining that linear analysis is an approximation for the relevant range of activities (Gonzalez, 2001; Uhlig, 2006; Drury, 2011). Therefore, the literature predominantly features linear models for multiproduct break-even analysis (Gonzalez, 2001; Kucharski and Wywiał, 2019; Wijayanti and Prasetyo, 2021). Within the framework of linear models, determination models (Dubonjić *et al.* 2016; Enyi, 2019) and CVP models of linear programming are distinguished (Kucharski and Wywial, 2019). Among deterministic models, the most significant ones are the conditional or standard product model and the average weighted contribution margin or profit model (Malinić *et al.* 2013; Stevanović and Petrović, 2016). CVP linear programming models can be based on different criteria, such as maximization of total revenue, total profit, or minimization of variable costs (Lazzari and Moriñigo, 2003; Kucharski and Wywiał, 2019).

Based on the established limitations and criticism of the assumptions of linear CVP models (Ndaliman and Bala, 2007; Kucharski and Wywiał, 2019), alternative approaches to calculating the break-even point in a multi-product situation were developed. A significant group of methods refers to nonlinear models (Wen-Hsien *et al*. 1990). Unlike linear models, nonlinear models of determining the multiproduct break-even point take into account nonlinear relationships between production, costs, and revenues. According to Horal *et al*. (2019), their results indicate that for certain types of activities, there isn't a single traditional break-even point but rather a range of production volumes that exhibit the principles of break-even activity. By applying the theory of averages and deviation analysis, they calculate the minimum and maximum break-even points. Additionally, there are studies that have developed mathematical models for dynamic break-even analysis considering the time value of money in multi-product production and have examined their application (Ngamsomsuke and Rabten 2022; Guang-bin and Bin-li, 2007).

In stochastic models, various types of uncertainties are distinguished, such as demand uncertainty (Asih and Eng, 2021), uncertainty in demand and return levels (Elfarou *et al.* 2022), uncertainty in the contribution margin and sales volume (Liang *et al.* 2021), uncertainty in production capacity planning (Asih and Eng, 2021), and others. Deterministic and stochastic models are often combined with optimization models. In optimization models, the multiproduct break-even point is determined based on different criteria of optimality, such as profit maximization (Briciu *et al.* 2013), minimization of production costs (Zhang *et al.* 2012), or multiple economic variables simultaneously (Elfarouk *et al.* 2022). Kucharski and Wywiał (2019) develop a model that aims to minimize variable production costs while considering the scale effects on production costs, as well as the model considers the stochastic aspect of business operations and aims to maximize the probability of profitability.

A considerable number of authors have attempted to address the multiproduct break-even point and uncertainty problem by applying a fuzzy logic-based determination system (Lazzari and Moriñigo, 2003; Fong-Ching, 2009; Konstantionos *et al.* 2009; Baral, 2016; Aslan and Yilmaz, 2018). The research conducted by Dohale *et al.* (2022) aimed at developing a multi-product multi-period (MPMP) aggregate production plan (APP) to fulfill customers' demand in terms of throughput and lead time, thus achieving market competence. This study proposes an integrated approach that combines the fuzzy analytical hierarchy process (FAHP), multi-objective linear programming (MOLP), and simulation. A multi-product multi-period (MPMP) aggregate production plan is applied to organizations that produce multiple products and have multiple planning periods.

In addition to these models, there are also CVP models based on various cost accounting systems. González (2001) utilized data from activity-based costing (ABC) systems, which track variable and fixed costs and require the model user to formulate a contribution rule for computing the necessary output of each product to achieve a target profit. In the study by Zhao and Yang (2022), research was conducted using the Time-driven activity-based costing (TDABC) system, relaxing the assumptions of the traditional CVP model. Furthermore, certain studies have combined contemporary cost accounting systems such as activity-based costing (ABC) with the theory of constraints (TOC). For example, Wen-Hsien *et al.* (2013) aimed to assess the integration of ABC and TOC, along with the application of a mixed-integer programming (MIP) model, to aid in decision-making regarding product mix using green manufacturing technologies (GMTs).

Lately, there has been a development in the behavioral approach which explores management behavior in the assessment segment of potential models, their limitations, and critical evaluation of their effects on decision-making (Martinović, 2019; Gubio *et al.* 2022). However, the most significant trend to be expected within the contemporary CVP approach relates to the utilization of various modern computer systems' performances. In the study by Thasan *et al*. (2023), an intelligent decision support system within the MCVP analysis framework was presented, with a created systemic platform to facilitate analysis.

The research gap identified in the explanation of linear models for multiproduct break-even analysis, based on the author's knowledge, pertains to comparing the solutions obtained from the break-even analysis model under different assumptions regarding the constancy of the proportions of individual products in relation to the contribution to physical output, total revenue, total variable costs, and contribution margin. It is necessary to analyze the solutions obtained from the break-even analysis model by using the conditional product model, the average weighted marginal contribution model, and solving systems of linear equations. Another aspect is verifying the optimality of the solutions obtained using the defined break-even analysis models.

Based on the previous discussion, we propose two research hypotheses:

H1: The break-even analysis model with constant ratios in a multiproduct scenario facilitates the achievement of optimal solutions.

H2: Optimal solutions of the linear programming model for multiproduct break-even point with constant ratios are identical to solutions obtained using models of relational equations.

To achieve the defined objectives, tasks, and hypotheses, the key methods will involve modeling and comparative analysis. The generalization of the analyzed models will primarily rely on mathematical proof methods and empirical verification.

3. Mathematical models of multiproduct break-even point

3.1. Definition of multiproduct break-even point

Assuming that the production/sales mix is predetermined and constant, the multiproduct break-even point represents the combination of production volumes for *n* individual products in a given period where the total revenue equals the total costs. The equation for the multiproduct break-even point is as in Equations (1) or (2).

$$\sum_{i=1}^{n}(p_i - w_i) \cdot q_i = F \tag{1}$$

$$\sum_{i=1}^{n} m_i \cdot q_i = F \tag{2}$$

In these equations, q_i is the production volume of the i-th product in a given period. p_i is the unit selling price of the i-th product. w_i is the unit variable cost of the i-th product. F is the total fixed costs in a given period. $m_i = p_i - w_i$ is the unit margin achieved by producing/selling the i-th product, $i = \overline{1,n}$.

In an *n*-dimensional vector space R^n, Equation (1) represents a hyperplane. Since $q_i \geq 0$ for $i = \overline{1,n}$, this hyperplane is constrained to a hyperpolygon whose extreme points intersect the n-coordinate axes and the hyperplane defined by Equation (1). The values of the extreme points of the hyper polygon located on the *i*-th coordinate axis can be calculated as $q_i^e = F/m_i$ for $i = \overline{1,n}$. This hyperpolygon exhibits the properties of a convex set and represents the set of feasible solutions (Zahirović *et al.* 2008; Winston, 2004). This means that every point belonging to this hyper polygon represents a possible multiproduct break-even point. In other words, any combination of production volume values $q_i \geq 0$, for $i = \overline{1,n}$, that satisfies Equation (1) represents a multiproduct break-even point. Therefore, the number of possible multiproduct break-even points is infinite.

To determine the number of suitable multiproduct decision points, additional constraints or assumptions need to be introduced. These commonly include considering a relevant range of activities and assuming a constant product mix (Malinić *et al.* 2013; Stevanović and Petrović, (2016). Production and market constraints can also be considered, among others.

In the models under consideration here, an additional assumption regarding the constancy of relationships has been introduced: the physical volume of production of individual products (Model 1), the share of total revenues of individual products (Model 2), the proportion of variable costs of individual products in total variable costs (Model 3), and the share of total margins of individual products (Model 4).

3.2. Constant proportions model of individual product physical volume

The coverage point model with constant relationships of physical production volume (Model 1) is based on the multiple proportion shown in Equation (3).

$$q_1 : q_2 : \cdots : q_n = k_1^v : k_2^v : \cdots : k_n^v \tag{3}$$

where k_i^v is the coefficient of proportionality for the production volume of the i-th product.

3.2.1. Determination model of the break-even point based on constant relationships of the physical production volume of individual products

By reducing the production volume of the *i*-th product to the production volume of a conditional (standard) product using the coefficients of proportionality from Equation (3), we obtain Equation (4).

$$q_i = k_i^v \cdot q \tag{4}$$

where q represents the production volume of the standard product. By substituting Equation (4) into (1) and solving for the unknown variable q, we obtain Equation (5).

$$q = \frac{F}{\sum_{i=1}^{n} k_i^v (p_i - w_i)} \tag{5}$$

Finally, the model of relational equations for the production volume of the *i*-th product at the break-even point is obtained by substituting Equation (5) into Equation (4), we obtain Equation (6).

$$q_i = \frac{k_i^v \cdot F}{\sum_{j=1}^{n} k_j^v (p_j - w_j)} \tag{6}$$

This formula can be used to calculate the break-even point under constant relationships of the physical production volume for individual products.

3.2.2. Optimization model with constant relationships of individual product production volumes

A linear programming model with constant relationships of individual product production volumes can have total revenue as the objective function, while the constraints are based on the fundamental break-even relation (1) and *n-1* constraints derived from Equation (3). The linear programming model is formulated as follows:

$$\text{Maximization: } R = \sum_{i=1}^{n} p_i\, q_i \tag{7}$$

Subject to:

$$\sum_{i=1}^{n} (p_i - w_i) \cdot q_i = F \tag{8}$$

$$k_{i+1}^{v} q_i - k_i^{v} q_{i+1} = 0 \text{ for } i = \overline{1, n-1}, \quad q_i \geq 0 \tag{9}$$

Since the linear programming model has *n* equations in the constraints, solving the system of *n* linear equations using one of the methods yields solutions for multiproduct break-even points that are equal to the solution given in Equation (6). In this *n*-dimensional point, the objective function reaches its minimum. This means that the solution obtained by the break-even analysis model is also an optimal solution, considering the assumption of constant relationships of individual product production volumes, which will be the subject of empirical testing.

3.3. Constant proportions model of total revenue for individual products

The model of the constancy of relationships of total revenues for individual products (Model 2) is based on the following multiple proportion which is Equation (10).

$$p_1 q_1 : p_2 q_2 : \cdots : p_n q_n = k_1^r : k_2^r : \cdots : k_n^r \tag{10}$$

where is k_i^r – coefficient of proportionality of the total revenue of the *i*-th product.

3.3.1. Determination model of the break-even point with constant relationships of total revenue for individual products

Using a similar procedure as in Model 1 the total revenue of the *i*-th product is reduced to the total revenue of the conditional (standard) product by utilizing the coefficients of proportionality from Equation (10), resulting in the following expression that is Equation (11).

$$p_i q_i = k_i^r pq \tag{11}$$

where pq represents the total revenue of the standard product at the standard price. Solving Equation (11) for q_i and substituting it into Equation (1) yields to Equation (12).

$$pq = \frac{F}{\sum_{j=1}^{n} \frac{k_j^r}{p_j} (p_j - w_j)} \tag{12}$$

By using relations (11) and (12), the expression for calculating the break-even point of the *i*-th product under constant relationships of the share of total revenues of individual products, we obtained a model of relational equation as in Equation (13).

$$q_i = \frac{\frac{k_i^r}{p_i} \cdot F}{\sum_{j=1}^{n} \frac{k_j^r}{p_j}\left(p_j - w_j\right)}, \text{for } i = \overline{1,n} \tag{13}$$

3.3.2. Optimization model with constant relationships of individual product total revenue

The linear programming model, with constant relationships of individual product total revenue, includes the same objective function (7) and constraint (8). The remaining *n-1* constraints are derived from the proportion (10) in the form of Equation (14).

$$k_{i+1}^r p_i q_i - k_i^r p_{i+1} q_{i+1} = 0 \text{ for } i = \overline{1, n-1}, \quad q_i \geq 0 \tag{14}$$

Similar to the previous model, the feasible solution domain is represented by a point in an *n*-dimensional space that is equal to the solution (13), which will be empirically tested.

3.4. Constant proportions model of total variable costs for individual products

The model of constant relationships of total variable costs for individual products (Model 3) is based on the following multiple proportion presented below as Equation (15).

$$w_1 q_1 : w_2 q_2 : \cdots : w_n q_n = k_1^w : k_2^w : \cdots : k_n^w \tag{15}$$

where k_i^w – the coefficient of proportionality of total variable costs for the i-th product.

3.4.1. Determination model of break-even point for constant ratios of total variable costs of individual products

By applying a similar procedure as in Model 2, the total variable costs of the *i*-th product are reduced to the variable costs of the conditional (standard) product using the coefficients of proportionality from equation (15), resulting in Equation (16).

$$w_i q_i = k_i^w wq \tag{16}$$

where wq represents the total variable costs of the standard product at the standard variable cost. By solving Equation (16) for qi and substituting it into Equation (1), we obtain Equation (17).

$$wq = \frac{F}{\sum_{j=1}^{n} \frac{k_j^w}{w_j}\left(p_j - w_j\right)} \tag{17}$$

By using relations (16) and (17), we obtain the model of relation equation for calculating the break-even point in the case of constant ratios of total variable costs of individual products are shown below in Equation (18).

$$q_i = \frac{\frac{k_i^w}{w_i} \cdot F}{\sum_{j=1}^{n} \frac{k_j^w}{w_j}(p_j - w_j)}, \text{ for } i = \overline{1,n}. \tag{18}$$

3.4.2. Optimization model of constant relationships of total variable costs of individual products

A linear programming model, with constant relationships of total variable costs of individual products, includes the objective function:

$$\text{Minimize: } W = \sum_{i=1}^{n} w_i\, q_i. \tag{19}$$

In addition to the constraint (1), *n-1* constraints would be introduced, obtained from the proportion (15), in the form of Equation (20).

$$k_{i+1}^{w} w_i q_i - k_i^{w} w_{i+1} q_{i+1} = 0 \text{ for } i = \overline{1, n-1}, \quad q_i \geq 0. \tag{20}$$

3.5. The model of constant relationships of the overall margin for individual products

This model is based on the assumption of the constancy of total marginal revenues for individual products (Model 4), and the multiple proportion can be written as Equation (21).

$$m_1 q_1 : m_2 q_2 : \cdots : m_n q_n = k_1^m : k_2^m : \cdots : k_n^m. \tag{21}$$

where is k_i^m – the coefficient of proportionality of the total margin of the *i*-th product.

3.5.1. Determination model of break-even point with constant proportions of total margins for individual products

Using a similar approach as in models 2 and 3, the total margin of the *i*-th product is transformed into the margin of the conditional (standard) product by utilizing the coefficients of proportionality from equation (21), resulting in Equation (22).

$$m_i q_i = k_i^m mq. \tag{22}$$

where mq is the total margin of the standard product with the standard margin. Through a similar procedure as in models 2 and 3, we obtain the model of relational equation for multiproduct break-even point in the case of constant proportions of total margins for individual products which brings us to Equation (23).

$$q_i = \frac{\frac{k_i^m}{m_i} \cdot F}{\sum_{j=1}^{n} \frac{k_j^m}{m_j}(p_j - w_j)}, \text{ for } i = \overline{1, n}. \tag{23}$$

3.5.2. Optimization model of constant relationships of total margins of individual products

A linear programming model, considering constant relationships of total margins of individual products, includes the objective function Equation (24).

$$\text{Maximize: } M = \sum_{i=1}^{n} m_i\, q_i \tag{24}$$

In addition to constraint (1), *n-1* constraints derived from the proportion (21) would be introduced in the following form of Equation (25).

$$k_{i+1}^{m} m_i q_i - k_i^{m} m_{i+1} q_{i+1} = 0 \text{ for } i = \overline{1, n-1}, \quad q_i \geq 0. \tag{25}$$

Based on the derived set of relational equations and the linear programming model, it can be observed that all developed multiproduct break-even point models with constant ratios exhibit characteristics of deterministic and linear models based on traditional cost accounting systems. Due to their generality, the developed models can be effectively applied to any multiproduct situation with stable market conditions and well-known operational environments, resulting in objective and precise determination of the break-even point.

4. Results and discussion

To test the analytical capabilities of the developed multiproduct break-even point models with constant ratios and to verify the set hypotheses, an empirical research methodology was applied based on the analysis of a selected case. This methodology involves selecting a specific company and constructing an analytical system related to the necessary input data.

4.1. Data

The chosen company operates in the metal processing industry and specializes in the production of containers. The primary reason for selecting this particular case was the convenience and accessibility of the necessary internal data (often considered proprietary), as well as assistance in rapidly assessing the operating conditions of the specific company.

The second reason is related to evaluating the fulfilment of assumptions of the linear and deterministic models with constant ratios developed in the study. These would be the following operating conditions of the selected company: a known a priori product mix with longer life cycles, production capacities, and applied technology allowing flexibility in production operations, the container market is not prone to sudden shocks and significant changes, thus input and finished product market prices are relatively predictable, the company's accounting system enables efficient cost classification, and the company has no extraordinary, financial, or capital revenues.

The company produces a total of seven models of metal containers, labelled from K1 to K7. Financial data has been extracted from the company's financial records. The financial and quantitative values pertain to a one-year period. The total fixed costs amounted to EUR 7,608,411. The total variable costs amounted to EUR 8,243,553. The total revenues amounted to EUR 17,137,259. Production volume from K1 to K7, respectively, amounted to 2,869; 2,294; 100; 284; 36; 390; and 100.

4.2. Results of hypothesis testing

In Table 1, it has been demonstrated that across all models of proportionality ratios, an optimal result exists. This establishes the precise multi-product break-even point for this enterprise.

Table 1. Model testing - multiproduct break-even point analysis

Description	Product type – metal container						
	K1	K2	K3	K4	K5	K6	K7
Unit selling price (EUR) - p_i	2,525	2,585	1,686	3,635	5,847	4,850	6,550
Unit variable costs (EUR) - w_i	1,263	1,230	1,440	2,019	2,045	1,949	2,209
Margin (EUR) - $m_i = p_i - w_i$	1,262	1,355	246	1,616	3,802	2,901	4,341
Model 1 k_i^v	79.69	63.72	2.78	7.89	1.00	10.83	2.78
Model 2 k_i^r	42.97	35.17	1.00	6.12	1.25	11.22	3.88
Model 3 k_i^w	147.18	126.36	1.00	18.66	5.56	45.99	17.65
Model 4 k_i^m	49.22	39.33	1.96	7.79	1.00	10.32	3.00
q_i (multiproduct break-even point)	2449	1958	85	242	31	333	85
q_i (optimal result)	2449	1958	85	242	31	333	85

Source: Author's own findings.

In the penultimate row of the preceding table, q_i (multiproduct break-even point), the results of calculating the multiproduct break-even point based on the relational models (6), (13), (18), and (23) are provided, which are identical for all models. The last row of the table, q_i (optimal result), contains the solutions of the linear programming models given in sections (3.2.2), (3.3.2), (3.4.2), and (3.5.2). The optimal solutions of all developed linear programming models are identical. It is evident that the solutions of the relational models and the linear programming models are identical.

In the case of a target function value of 14,621,192 EUR, representing the minimum revenue required to cover costs, all four models have shown that the optimal multi-product product structure with specified production quantities is identical. The quantity of products per product type is presented in the table (q_i): 2449 (K1), 1958 (k2), 85 (K3), 242 (K4), 31 (K5), 333 (K6) and 85 (K7). Total margin at the multiproduct break-even point is EUR 7,608,590. Total value of variable costs at the multiproduct break-even point is EUR 7,012,602.

4.3. Discussion of research results and research limitations

In this study, which evaluated linear models of multiproduct break-even analysis, it has been confirmed that these models, despite being known for several decades, can still be researched, and improved upon. The models of relational equations based on weighted-average unit contribution margin are already known in the literature (Potkany and Krajcirova, 2015; Hilton, 2008). However, in this study, we demonstrated that it is possible to generalize the modeling of the multiproduct break-even point with constant ratios, considering each of the individual elements of the multiproduct break-even point: the physical production volume, total revenue, variable costs, and individual product margins. Although the models of linear programming for multiproduct break-even points are also known in the literature (Kucharski and Wywial, 2019), the generalized approach presented in this study has shown that it is possible to develop linear programming models considering all potential criteria relevant to multiproduct situations: the physical production volume, total revenue, variable costs, and individual product margins.

It has been demonstrated that solutions of linear models analyzing the profitability of multiple products with constant contribution structures have consistent effects, irrespective of their nature. Based on empirical research, the key finding is that all developed models of relational equations and linear programming models based on constant ratios provide identical solution

s. Thus, any of the models developed in the study can be used for calculating the multiproduct break-even point depending on the available data. Consequently, the research hypotheses are empirically confirmed. This implies that multiproduct break-even point models with constant ratios enable obtaining optimal solutions, regardless of whether relational or optimization models are used, and that the solutions of linear programming models and models of relational equations are identical. This suggests that the derived formulas can be used regardless of the assumption of the constancy of individual product contribution structures, thereby expanding the theoretical and methodological insights of deterministic and linear models based on traditional cost accounting (Potkany and Krajcirova, 2015; Hilton, 2008; Kucharski and Wywial, 2019).

Although our focus in obtaining solutions was on the physical production volume of individual products in a multiproduct break-even point, the results can easily be extended to the financial aspect. This includes calculating total revenue, variable costs, and contribution margin both per product and overall. These enhancements in the models contribute to understanding and analyzing multiproduct break-even points, enabling more comprehensive decision-making and planning in the context of production and financial management.

However, it is important to acknowledge the limitations of this research. The study focused on linear models of profitability analysis for multiple products with constant contribution structures. The findings may not be directly applicable to cases with different contribution structures or nonlinear relationships (Martinović, 2019). Additionally, the research assumed constant ratios and did not account for potential fluctuations or uncertainties in market conditions, cost structure, or demand. Future research could explore more complex scenarios and consider dynamic factors to provide a more comprehensive understanding of multiproduct profitability analysis.

5. Conclusion

This scientific paper investigated and analyzed linear models of the multiproduct break-even point with constant relationships between total variable costs and the total margin of individual products. The research results confirm that these models represent optimal solutions and are applicable in practical contexts. The key contribution of this research lies in providing clear formulas and models for decision-making regarding the utilization of production capacities, selection of the production mix, and determination of the production volume for individual products. Their practical application enables companies to efficiently manage their production processes and optimize their profitability. The implementation of these models demonstrates that they are user-friendly and interpretable, making them valuable tools for decision-making in a business environment. Their application allows companies to assess the current business situation, predict the impact of different decisions on profitability, and identify optimal strategies to achieve desired outcomes.

The significance of this scientific paper is multifaceted, encompassing practical applicability, enhancement of the analytical foundation, and contribution to the theoretical understanding of the multiproduct break-even point. Through the application of linear models and analytical approaches, the research builds upon existing literature and theory in this field. By solving specific problems and testing the models on a practical example of metal container production, the research confirms that previously developed models and formulas can be successfully applied in real-life situations. Overall, the results of this research contribute to the understanding and application of linear models for decision-making in the context of multiproduct break-even analysis. The simplicity and effectiveness of these models make them valuable tools for businesses in assessing their production strategies and optimizing their profitability.

Further research could explore the applicability of these models to other types of production environments, such as heterogeneous production or nonlinear relationships between variables. Additionally, incorporating risk and uncertainty factors into the models could provide a more comprehensive decision-making framework. Multidisciplinary research encompassing the fields of accounting, behavioral economics, computer science, operations research, and information systems is required. Special focus could involve the most intelligent trends in machine learning and artificial intelligence.

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